

# **OPTIMAL MANAGEMENT OF MULTIRESERVOIR WATER RESOURCES SYSTEMS**

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**MASTER OF TECHNOLOGY**

by  
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to the  
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## CERTIFICATE

This is to certify that the thesis 'Optimal Management of Multireservoir Water Resources Systems' submitted by Shri K.V. Jayakumar, in partial fulfilment of the requirements for the degree of Master of Technology of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under my supervision and guidance. The work embodied in this thesis has not been submitted elsewhere for a degree.



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## LIST OF SYMBOLS

$a_{ij}$	Arc connecting node $i$ to node $j$
$b_j$	Least square regression coefficient for estimating $(j+1)$ th flow from $j$ th flow
$b_{ij}$	Cost of passing one unit of flow through arc $(i,j)$
$D_{jk}$	Demand at node $j$ in time period $k$
$f_{ij}$	Flow in arc $(i,j)$
$g$	Unbiased estimate of population skew coefficient
$H_{ij}$	Upper bound on arc $a_{ij}$
$i$	Month number
$I_k$	Rate of import of water in time period $k$
$K$	Normal standard deviate
$K'$	Monthly flow logarithm expressed as normal standard deviate
$K_t$	Correlation component of streamflow
$k$	Time period
$L_{ij}$	Lower bound on arc $a_{ij}$
$l_{ij}$	Lower flow capacity of arc $(i,j)$
$m$	Year number
$N$	Total years of record in eqn. (2.6) to eqn. (2.8)
	Number of nodes in the network in AL-3 model
$n_t$	Number of reservoirs plus link junctions in each period
$n_r$	Number of reservoirs
$P_{jk}$	Rate of spill from reservoir or link junction $j$ in time period $k$

$Q$	Monthly recorded streamflow
$Q_{ijk}$	Flow between reservoir or link junction $i$ and reservoir or link junction $j$ in time period $k$
$q$	Small increment of flow used to prevent infinite negative values of logarithm of incremental monthly flow for months of zero flow
$q_{ij}$	Total cost of transporting one unit of flow from node $i$ to node $j$ in OKA  Flow from node $i$ to node $j$ in AL-3 model
$S$	Unbiased estimate of population standard deviation
$S_{jk}$	Storage contents of reservoir $j$ in time period $k$
$S_t$	Periodic or seasonal component of streamflow
$s$	Standard deviation of annual flows
$T$	Number of time periods
$T_t$	Trend component of streamflow
$t$	Pearson Type III standard deviate
$t_i$	Normal random variate with mean zero and variance unity
$U_{ij}$	Upper bound on flow from node $i$ to node $j$
$X$	Logarithm of incremental monthly flow
$\bar{X}$	Mean of logarithm of incremental monthly flows
$X_i$	Annual runoff for $i$ th year
$X_t$	Streamflow
$\alpha_{jk}$	Input to reservoir $j$ in time period $k$
$\beta_{jk}$	(Demand+evaporation) from reservoir $j$ in time period $k$
$\gamma_1$	Annual lag one serial correlation coefficient



$\delta_j$	1 if $j$ is an import node 0 if $j$ is not an import node
$\Delta t$	Unit of each time period
$\varepsilon_t$	Random component of streamflow
$\pi_j$	Price of one unit of water at the node $j$
$\theta_j$	1 if $j$ is spill node 0 if $j$ is not a spill node

## ABSTRACT

Water Resources Systems are complex. Models are used for design and operation of such complex systems. The models may be physical or mathematical. In the present study, a mathematical model is used for analysis of such systems. A set of simulation and optimization techniques used for analyzing the models of multireservoir water resources system is explained. A resume of the development of synthetic streamflow generating techniques is given. Making use of an existing program for streamflow generation, synthetic streamflows are generated for two neighbouring river basins. For optimization studies, the concept of representing the water resources system in a capacitated network form is made use of. The out-of-kilter algorithm which makes use of this concept is explained. An existing program, developed by the Texas Water Development Board, which makes use of the out-of-kilter algorithm is used to study the performance of the multi-reservoir system in one of the basins. The results of the simulation and optimization are presented and discussed.

## CHAPTER 1

### INTRODUCTION

#### 1.1 General

Water is one of the several resources without which a nation cannot satisfy the fundamental wants of its people or achieve the important national goals it sets for itself. Without water, life itself cannot be sustained. Our nation is blessed with a bountiful supply of water, although it is not always in the right place at the right time. Because of the general abundance of water, it was taken for granted that water had no cost and there were no limits to its availability. But as demands come close to and in some regions even exceed supplies of water, it becomes necessary to seek ways to increase the efficiency in the use of water. The scope of this study is to optimally manage a complex water resources system involving a system of multiple reservoirs. The study would also reveal surpluses or shortages that would occur in the years of high or low flows compared with years of mean flows.

#### 1.2 Literature Review

The application of computer oriented optimization techniques to solve design and operation problems in the field of water resources development has received serious attention and support in the past few years. These techniques may be divided into two

main categories : simulation and mathematical programming.

Simulation has been effectively used in several investigations to find the best feasible design for a complex multi-purpose multi-reservoir system (Maass, 1962). Hufschmidt and Fiering (1966) have used the simulation technique to analyse Lehigh and Delaware water resources systems.

The Lehigh Water-Resource System consisted of a system of six reservoirs out of which only five could be built at any time. Each reservoir was a multipurpose reservoir and could be used to provide (i) regulated flows for water supply or water quality improvement, (ii) recreation, (iii) flood protection, and (iv) hydro electric energy. The system also included a diversion channel. The simulation model of the Lehigh system had 42 major design variables.

In these studies, a number of equations are formulated to describe the physical behaviour of the water resources system under investigation. Design variables (capacities of reservoirs, power plants, and irrigation canals) and operating rules (monthly release from reservoirs as a function of reservoir content and month of the year) are introduced into these equations as parameters. The problem is run through the computer for many combinations of the parameters. Physical feasibility of each design and the value of the objective function are determined by the computer. The design with maximum value of

the objective function is chosen as the optimum design. These studies have shown that simulation technique perform satisfactorily.

The introduction of the concept of reservoir zoning by Beard (1967), Fredrich and Beard (1972) was a significant development in the simulation technique. In this concept, every reservoir is divided into a number of storage zones and during simulation, all reservoirs are maintained in the same zone as far as possible. A generalized simulation program for the operation of a reservoir system for ~~conservation~~ purposes such as water supply, navigation, recreation, low flow augmentation and hydro-electric power has been developed by the Hydrologic Engineering Centre (1974). Visens and Schaake(1980) have studied the Rio Colorado basin using the simulation technique.

Another major development in simulating complex water resources systems was the application of out-of-kilter algorithm to such systems (Ford and Fulkerson, 1962). The technique has been used by the Texas Water Development Board in a number of their programs (TWDB-1, 1970, TWDB-2, 1972).

Mathematical programming techniques are analytical methods with some theoretical assurance that the optimal solution will be reached given enough computer time. Some of the important works done using these techniques are reported in Chapter 3. In

addition to those, Harboe et al. (1970) used the dynamic programming to develop optimal policy for reservoir operation, Torabi and Mobasheri (1973) and Tauze et al. (1980) have also used dynamic programming for optimization studies. Windsor and V.T. Chow (1972) used integer programming and separable programming to develop a multi-reservoir optimization model. Himmelblau (1974) and Bayer (1974) used non-linear programming in their studies.

In the present study, a monthly simulation program HEC-4 developed by Leo R. Beard (1972) of the Hydrologic Engineering Centre, is used to generate synthetic streamflows. An optimization model AL-3, which makes use of the out-of-kilter algorithm developed by the Texas Water Development Board is used with substantial modifications.

### 1.3 Organization of the Report

In Chapter 2 of this thesis, the development of the techniques of synthesis of streamflow is summarised and the description of the streamflow generation model adopted is given. The description of the out-of-kilter algorithm is given in Chapter 3 along with an illustrative example and the model used in the present study is presented. Chapter 4 contains the descriptions of the basins selected for study and the analysis of the river basin system. The results of the study are presented and discussed in Chapter 5.

#### 1.4 Units Used in the Study

In order to keep the study as close to a real problem as possible, actual data from published literature for certain river basins in India were used. These data are generally in F.P.S. system and hence the same units are retained in the study.

## CHAPTER 2

### STREAMFLOW GENERATION

#### 2.1 Introduction

A common constraint encountered in water resources management is the inadequacy of streamflow records. If policy decisions on design and operation of water resources system are based on inadequate information, the response of the system under a full range of conditions cannot be postulated. If the length of streamflow record is short, critical sequences of years of low and high runoff inherent in the statistical population of river flows may be missing. But no matter how poorly a brief record may identify the time frequency of years or seasons of unusually high or low flows, unless it is very short indeed, it will permit fairly reliable estimates of mean annual and seasonal flows and their variances. These statistical parameters, together with a few assumptions about the population of flows, can make it possible to construct statistical models that can generate synthetic flows of any desired length (Maass et al., 1962).

#### 2.2 Time Series Components of Streamflow Data

From a statistical point of view, streamflow data can be regarded as consisting of four components (Kottegoda, 1970) viz., trend  $T_t$ , periodic or seasonal  $S_t$ , correlation  $K_t$  and



random  $\varepsilon_t$  components which can be combined simply as follows

$$X_t = T_t + S_t + K_t + \varepsilon_t \quad (2.1)$$

A sequence of values arranged in order of their occurrence is called a time series. A time series is considered to be stationary if the statistical properties characterising it are time invariant. The non-stationary data can be made stationary by a simple transformation (McMohan and Mein, 1978).

One characteristic of a time-series is persistence which relates to the sequencing of the data. In streamflow, persistence arises from natural catchment storage effects which tend to delay run-off; over a short period of time, high flows in one interval will tend to be followed by high flows in the following interval. The longer the time period, the lesser the effect and for many streams it is negligible for annual flows (McMohan and Mein, 1978).

The usual quantitative measure of persistence is serial correlation. Serial correlation coefficients may be calculated for the correlation between the flow in any given time period (for example, month or year) and the flow in  $k$  time periods earlier where  $k$  is called the lag ( $k = 1, 2, \dots$ ). In many studies, only the lag one serial correlation is considered, that is, the persistence between an event and the

immediately preceding event. Lag one models have been shown to be operationally satisfactory in several studies (Kottogada, 1970).

### 2.3 Development of Synthetic Streamflow Generation Techniques

Allen Hazen (1914) is considered to be the first to recognize the desirability of extending hydrological data. He synthesized a runoff sequence of 300 years by combining annual-mean-flow series for 14 streams in which the flows of each were expressed in terms of individual mean flows. This method in effect combines samples from different populations and is thus not precisely applicable to any particular stream. Charles E. Sudler (1927) employed a deck of 50 cards, on each of which was printed a representative annual streamflow. By dealing this deck 20 times, he obtained an artificial record of 1000 years. The adequacy of this method depends on how the values printed on the deck are determined. Furthermore, the method has the unrealistic limitation that the largest flow in 50 years is also the largest flow for the entire record. In this method, all periods of 50 years have the same mean, the same standard deviation and the same range which is a major defect. F.B. Barnes (1954) used a similar method to that of Sudler except that the synthetic flows were approximately made normal variates with the same mean and standard deviation as the flows of the historical record. Barnes introduced the improvement of using a table of random numbers

in synthesizing a 1000 year sequence of streamflows. The use of Barnes method is limited to the representative annual flows of a single stream that are approximately normally distributed and that do not exhibit serial correlation.

Markov introduced the concept of a process in which the probability distribution of the outcome of any trial depends only on the outcome of the directly preceding trial and is independent of the previous history of the process. In this 'trial' is the passage of one year and its 'outcome' is the streamflow for that year. If the probability distribution of annual streamflow is either independent of previous streamflows or correlated with only the previous year flow, we have a 'simple' or 'lag one' Markov process. The concept has been extended to include cases of lag greater than one and the process has been the basis of study and developments of streamflow generation procedures during early sixties (Maass, 1962).

Brittan proposed the following Markov model to represent actual streamflows (McMohan and Mein, 1978).

$$X_{i+1} = \mu + \gamma_1(X_i - \mu) + t_i s (1 - \gamma_1^2)^{\frac{1}{2}} \quad (2.2)$$

where

$X_i, X_{i+1}$  = annual run offs for  $i$ th and  $(i+1)$ th year,

$\mu$  = mean historical annual flows,

$s$  = standard deviation of annual flows,

$\gamma_1$  = annual lag one serial correlation coefficient,  
and

$t_i$  = normal random variate with mean of zero and a variance of unity.

This model consists of two components : a deterministic or correlation component  $[\mu + \gamma_1(X_i - \mu)]$  and a random component  $[t_i s(1 - \gamma_1^2)^{\frac{1}{2}}]$ .

In the annual Markov model outlined above, only two of the four components assumed to make up the streamflow process, as defined in equation (2.1) are accounted for explicitly. Trend and periodicity are not considered.

The most common form of periodicity relates to seasonality, particularly with respect to monthly flow generation. Here, the most appropriate practical model is the one proposed by Thomas and Fiering (1962). The algorithm for the Thomas and Fiering seasonal model is as follows :

$$X_{i+1} = \mu_{j+1} + b_j(X_i - \mu_j) + t_i S_{j+1}(1 - \gamma_j^2)^{\frac{1}{2}} \quad (2.3)$$

where

$X_{i+1}, X_i$  = generated flows during (i+1)th and ith seasons reckoned from the start of the synthesized sequences.

$\mu_{j+1}, \mu_j$  = mean flows during (j+1)th and jth seasons within a repetitive annual cycle of seasons (if months are being used,  $1 \leq j \leq 12$ )

$b_j$  = least square regression coefficient for estimating (j+1)th flow from jth flow

$$b_j = \gamma_j \frac{S_{j+1}}{S_j} \quad (2.4)$$

$t_1$  = normal random deviate with mean of zero, and variance unity

$S_{j+1}, S_j$  = standard deviations of flows during (j+1)th and jth seasons, and

$\gamma_j$  = correlation coefficient between flows in jth and (j+1)th seasons.

To use the model to generate monthly flows at a site, 36 parameters - monthly means, standard deviations and lag one serial correlation - are required. These are obtained from analysis of monthly historical flows.

This model is restricted to normally distributed flows, that is,  $t_1$  is considered to be a normal random deviate. In order to cater for non-normal streamflows, the model can be modified by any of the following alternatives.

- (1) modify  $t_1$  by an appropriate transformation
- (2) modify the streamflow parameters and the model algorithms such that the final generated data are distributed like the historical flow upon which they are based
- (3) generate normally distributed flows and apply inverse normalizing equations.

Matalas (1967) presented moment transformation equations which theoretically preserve the moments and lag one serial correlation coefficients. This method assumes that the logarithms of the flows are normally distributed. The procedure

is first to calculate a series of logarithms using a normal model and then obtain absolute flows by exponentiation.

#### 2.4 Method Used in the Present Study

In the present study, a method developed by Beard(1972) of the Hydraulic Engineering Centre, U.S. Army Core of Engineers, for multi-sites and multi-periods is used. The following equations are given by him.

$$X_{i,m} = \log(Q_{i,m} + q_i) \quad (2.5)$$

$$\bar{X}_i = \frac{\sum_{m=1}^N X_{i,m}}{N} \quad (2.6)$$

$$S_i = \sqrt{\frac{\sum_{m=1}^N (X_{i,m} - \bar{X}_i)^2}{(N-1)}} \quad (2.7)$$

$$g_i = N \frac{\sum_{m=1}^N (X_{i,m} - \bar{X}_i)^3}{[(N-1)(N-2)S_i^3]} \quad (2.8)$$

where

$X$  = logarithm of incremental monthly flow,

$Q$  = monthly recorded streamflow,

$q$  = small increment of flow used to prevent infinite negative values of  $X$  for months of zero flow,

$\bar{X}$  = mean logarithm of incremented monthly flows,

$N$  = total years of record,

$S$  = unbiased estimate of population standard deviation,

$g$  = unbiased estimate of population skew coefficient,

$i$  = month number, and

$m$  = year number.

Each individual flow is then converted to a normalized standard variate, using the following approximation of the Pearson Type III distribution.

$$t_{i,m} = (K_{i,m} - \bar{X}_i) / S_i \quad (2.9)$$

$$K_{i,m} = \frac{6}{g_i} [ ((g_i t_{i,m} / 2) + 1)^{1/3} - 1 ] + \frac{g_i}{6} \quad (2.10)$$

where

$t$  = Pearson Type III standard deviate, and

$K$  = normal standard deviate.

The above equations are used for generation. The generated normal standard deviates are converted back to flows by use of the following equations.

$$t_{i,m} = [ ((g_i/6)(K'_{i,m} - g_i/6) + 1)^3 - 1 ] 2/g_i \quad (2.11)$$

$$K_{i,m} = \bar{X}_i + t_{i,m} S_i \quad (2.12)$$

$$Q_{i,m} = -q_i + \text{Antilog } K_{i,m} \quad (2.13)$$

and

$$Q_{i,m} \geq 0 \quad (2.14)$$

where

$K'$  = monthly flow logarithm expressed as a normal standard deviate.

## 2.5 Streamflow Generation

Generation of streamflows is accomplished by starting with average values for all stations in the first month and

discarding the first two years of generated flows to ensure a really random start. Maximum, minimum and average flows are obtained for the entire period of flows as recorded and for specified periods of years.

Because of limitations in computer memory size and also due to increasing change of computational instability with larger matrices, the number of stations usable simultaneously in this model has been limited to eight. By including a few important stations from one set to the next set of stations, again limiting the total to eight, simultaneous flows for all the stations in a basin is generated, preserving the important correlations.



## CHAPTER 3

### OPTIMIZATION MODEL

#### 3.1 General

Water resources problems are becoming more complex and larger in size. Their efficient planning and design requires that the most powerful analytical techniques available be used. Simulation and optimization techniques coupled with high speed digital computers can, if used properly, provide the planner with valuable decision-aiding information. In this chapter, a brief description of the model used in the present study is given.

#### 3.2 Techniques for Optimization of a Water Resources System

Linear programming and dynamic programming are two of the optimization techniques that have been widely used in water resources system optimization studies. Linear programming has been used to solve many water resources problems by researchers like Rogers (1969), Marglin (1962), Maass (1962). Thomas and Revelle (1966) and Loucks (1969). The use of dynamic programming for solving water resources problems was pioneered by Hall and his co-workers (Hall and Buras, 1961; Hall, 1964; Hall and Howell, 1963; Hall and Roefs, 1966; Hall, Butcher and Esogbus, 1968). Later, Mobasheri and Harboe (1970) and Dutcher and Sunder (1973) also used the dynamic programming technique for optimization studies.

In the present study, an optimization technique which makes use of the out-of kilter algorithm (OKA) introduced by Fulkerson (1961) is used. Himmelblau and O'Laoghaire (1974) have demonstrated the use of this algorithm in solving a water resources optimization problem.

### 3.3 Out-of-Kilter Algorithm

This algorithm makes use of the concept that the water resources system can be represented by a series of nodes and arcs in a capacitated network form analogous to electric circuit. The problem solved by OKA is essentially a linear programming problem with the special feature that a number of equality constraints exist. Computational results for some large scale problems show OKA to produce a solution in one twentieth to one fiftieth the time of standard linear programming codes (Himmelblau and O'Laoghaire, 1974). This is due to the following reasons :

- 1) all operations are additive (i.e., no multiplication or decision takes place)
- 2) no matrix inversion is necessary.

The use of OKA to solve a water resources problem is illustrated by an example taken from Himmelblau (1974). The problem selected is a minimum cost circulation problem. The various symbols used in this example are shown in Table 3.1. The nine possible mutually exclusive 'kilter conditions' for each arc as the algorithm proceeds to seek an optimal solution is shown in Table 3.2.

Table 3.1 Symbols Used in the Out-of-Kilter Algorithm

---

$b_{ij}$	=	Benefit of passing one unit of flow through arc (i,j)
$f_{ij}$	=	Flow in arc (i,j)
$l_{ij}$	=	Lower flow capacity of arc (i,j)
$q_{ij}$	=	Total cost of transporting one unit of flow from node i to node j
$u_{ij}$	=	Upper flow capacity of arc (i,j)
$\pi_j$	=	Price of one unit of water at the node j

---

Table 3.2 Possible kilter-conditions for an arc

---

State	$q_{ij}$	$f_{ij}$	In Kilter?
A	$q > 0$	$f = l$	Yes
B	$q = 0$	$l \leq f \leq u$	Yes
C	$q < 0$	$f = u$	Yes
$A_1$	$q > 0$	$f < l$	No
$B_1$	$q = 0$	$f < l$	No
$C_1$	$q < 0$	$f < u$	No
$A_2$	$q > 0$	$f > u$	No
$B_2$	$q = 0$	$f > u$	No
$C_2$	$q < 0$	$f > u$	No

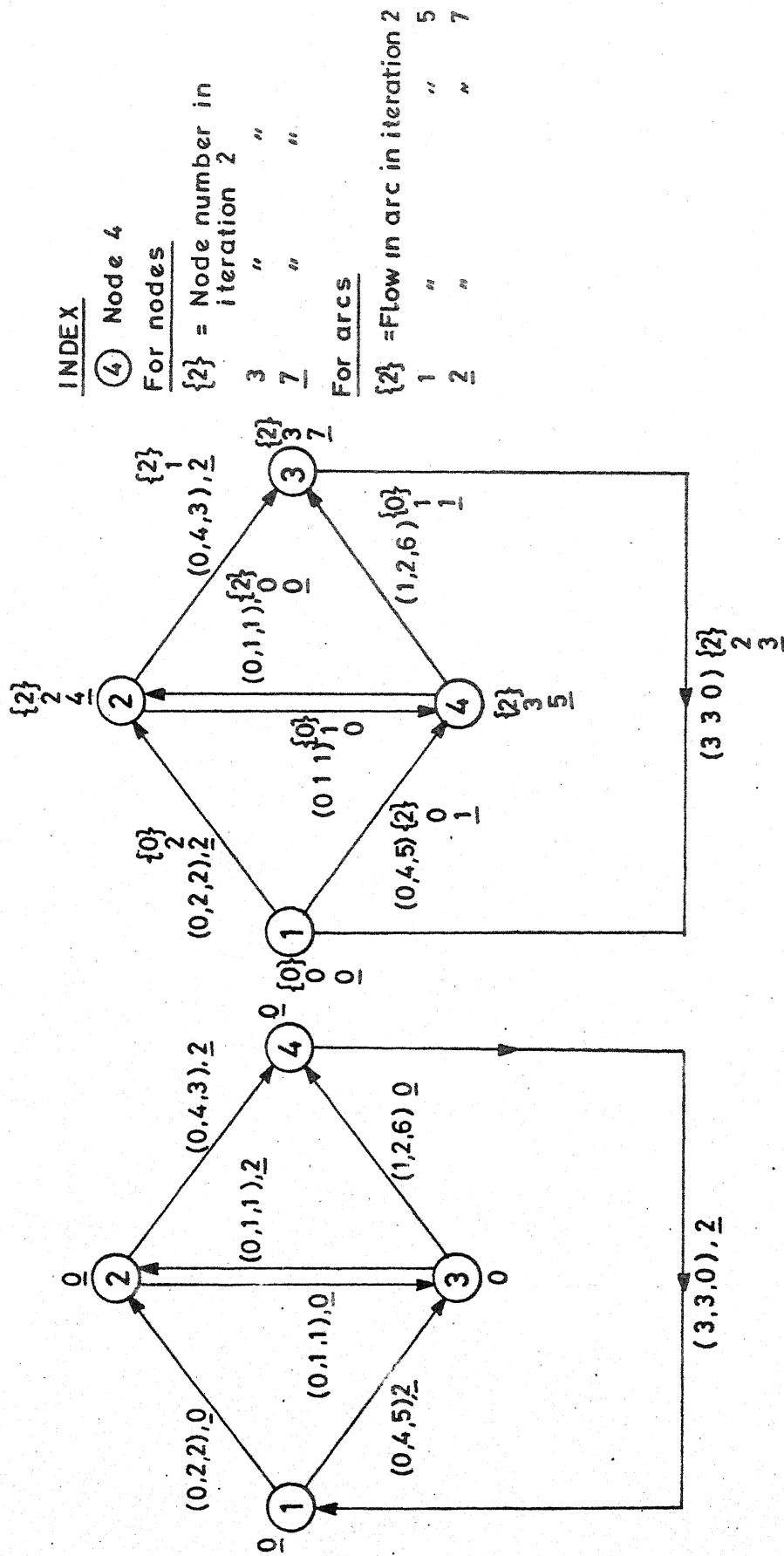
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Fig. 3.1 shows the network representation of the example. The ordered triple  $(\underline{l}_{ij}, \underline{u}_{ij}, \underline{b}_{ij})$  is shown on each arc. The original arc flow and node numbers ( $\pi$  values) appear as underlined numbers. Arc  $(1,3)$  is out of kilter and in state  $A_2$ . This arc can be brought into kilter by changing flows in the closed path  $(1,2), (3,2), (1,3)$ , by increasing flows in forward arcs and decreasing flows in reverse arcs. The process is continued till all the arcs are brought into kilter. The node numbers ( $\pi$  values) keep changing with each iteration and a labelling procedure is to be followed.

### 3.3.1 Labelling procedure

1. If an arc  $(i,j)$  in state  $A_1, B_1$  or  $C_1$  is brought into kilter by increasing the flows, then the node  $j$  is labelled  $(i^+, e(j))$ . This means that node  $j$  may receive  $e(j)$  units from node  $i$ .  $e(j)$  is taken to be  $(\underline{l}_{ij} - f_{ij})$  if the arc is in state  $A_1$  and  $(\underline{u}_{ij} - f_{ij})$  if in state  $B_1$  or  $C_1$ .
2. If an arc  $(i,j)$  in state  $A_2, B_2$  or  $C_2$  is brought into kilter by decreasing the flows, then the node is labelled  $(j^-, e(i))$  meaning that the flow from node  $i$  to node  $j$  can be reduced by  $e(i)$ .  $e(i)$  is taken to be  $(f_{ij} - \underline{l}_{ij})$  if the arc is in state  $A_2$  and  $(f_{ij} - \underline{u}_{ij})$  if in state  $B_2$  or  $C_2$ .

An arc in any of the possible remaining states  $A, B$  or  $C$  is in kilter and its flow should not be changed.



(a) Example problem (b) Flow values and node numbers in iterations 2,5 & 7.

Fig.3.1 Minimum cost circulation problem

### 3.3.2 Solution

Suppose that arc  $(i,j)$  is out of kilter and node  $i$  has been labelled. Now, the aim is to find a flow augmenting path from  $i$  to  $j$  in such a way that in-kilter arcs in the path will not be driven out of kilter. An arc that originates or terminates at a labelled node is considered and an attempt is made to label the node at the arc's connecting end. If node  $j$  is labelled, a flow augmenting path has been found and the flow in connecting cycle is changed according to the label value. Thus arc  $(i,j)$  is brought either into kilter or made less out of kilter. Now, another out-of-kilter arc is selected and the procedure is repeated.

The five principle steps of the OKA are given below.

- 1) Find an out-of-of kilter arc  $(i,j)$ . If none, the optimal solution has been found.
- 2) Determine whether the flow in the arc should be decreased or increased to bring the arc into kilter.
- 3) If the flow in the arc is to be decreased, find a path from  $i$  to  $j$  along which the flow can be increased without causing any arc to become out-of-kilter. Increase the flow in the path and decrease the flow in  $(i,j)$ . If  $(i,j)$  is now in kilter, go to step 1. If  $(i,j)$  is out of kilter repeat step 3. If no path is found go to step 5.

- 4) If the flow in the path is to be increased, find a path from  $j$  to  $i$  along which the flow can be increased without causing any arc to become out-of-kilter. Increase the flow in the path and also in  $(i,j)$ . If  $(i,j)$  is now in kilter, go to step 1. If  $(i,j)$  is out of kilter, repeat step 4. If no path is found go to step 5.
- 5) Change the  $\pi$  values and repeat 2 for arc  $(i,j)$ , keeping the same labels on all nodes already labelled. If the node numbers become infinite, no feasible solution is possible.

Following these five steps, the example problem is solved. Seven iterations are necessary to obtain the optimal solution. The results of the first iteration is shown in Table 3.3. The final results, when all the arcs are brought into kilter is shown in Table 3.4. The Fig. 3.1(b) shows the results of 2nd, 5th and 7th iterations.

A small network with 4 nodes and 7 arcs takes 7 iterations. In complex water resources system, where the number of nodes and arcs are larger and where steps 3 and 4 of this article are repeated, the number of iterations will be enormous and a high speed digital computer becomes essential to solve such problems.

Table 3.3 Results of Iteration 1

Arc (i,j)	$\pi_i$	$-\pi_j$	$-b_{ij}$	$q_{ij}$	$f_{ij}$	State	In kilter?
(1,2)	0	0	2	2	$0 = 1_{12}$	A	Yes
(1,3)	0	0	5	5	$2 > 1_{13}$	A <sub>2</sub>	No
(2,3)	0	0	1	1	$0 = 1_{23}$	A	Yes
(2,4)	0	0	3	3	$2 > 1_{24}$	A <sub>2</sub>	No
(3,2)	0	0	1	1	$2 > 1_{32}$	A <sub>2</sub>	No
(3,4)	0	0	6	6	$0 < 1_{34}$	A <sub>1</sub>	No
(4,1)	0	0	0	0	$2 < 1_{41}$	B <sub>1</sub>	No



Table 3.4 Results of Iteration 7

$(i, j)$	$\pi_i$	$-\pi_j$	$-b_{ij}$	$q_{ij}$	$f_{ij}$	State	In kilter?
,2)	0	-4	2	-2	$2 = u_{12}$	C	Yes
,3)	0	-5	5	0	$1_{12} \leq 1 \leq u_{12}$	B	Yes
,3)	4	-5	1	0	$0 = 1_{23}$	B	Yes
,4)	4	-7	3	0	$1_{24} \leq 2 \leq u_{24}$	B	Yes
,2)	5	-4	1	2	$0 = 1_{32}$	A	Yes
,4)	5	-7	6	4	$1 = 1_{34}$	A	Yes
,1)	7	-4	0	7	$3 = 1_{41}$	A	Yes

### 3.4 Description of the Model Used

An Allocation Model, AL-3, developed by Texas Water Development Board (TWDB-2, 1972) is used. This is a general hydrologic model of surface water resource systems. The model is designed to analyze the simulated multiperiod operation of any interconnected configurations of reservoirs, pump canals and pipe lines on a steady state monthly or seasonal basis. Substantial modifications are made in this model to suit the present requirements and also to improve the efficiency of the model in redistributing the deficits evenly to various demand nodes.

#### 3.4.1 Mathematical model

The out-of-kilter algorithm, OKA (Himmelblau, 1974) used in the model solves the following problem :

For the arcs and nodes defined in the system,

$$\text{minimize} \quad \text{cost} = \sum_i \sum_j b_{ij} q_{ij} \quad (3.5)$$

$$\text{subject to} \quad q_{ij} \leq H_{ij} \quad (3.6)$$

$$q_{ij} \geq L_{ij} \quad (3.7)$$

$$\sum q_{ij} - \sum q_{ji} = 0 \quad \text{for each } i \quad (3.8)$$

and

$$q_{ij} \geq 0 \quad \text{for each } a_{ij} \quad (3.9)$$

where  $b_{ij}$  = cost of passing one unit of flow through arc  $a_{ij}$ ,

$q_{ij}$  = quantity of flow passing through arc  $a_{ij}$

$H_{ij}$  = upper bound on arc  $a_{ij}$ ,

$L_{ij}$  = lower bound on arc  $a_{ij}$ ,

and

$a_{ij}$  = arc connecting node  $i$  to node  $j$ .

Eqn. (3.8) when applied to the various nodes in the network gives the following equations. The various terms used in the equations are defined in the Table 3.5.

(1) Initial storage node :

$$\sum_{j=1}^{n_r} \left[ \frac{S_{ji}}{\Delta t} \right] = X_c \quad (3.10)$$

(2) Input node :

$$\sum_{k=1}^T \sum_{j=1}^{n_r} \alpha_{jk} = X_i (\text{Inflow from Source Node}) \quad (3.11)$$

(3) Demand node :

$$\sum_{k=1}^T \left[ \sum_{j=1}^{n_r} \beta_{jk} + \sum_{j=n_r+1}^{n_t} D_{jk} \right] = X_d \quad (3.12)$$

(Outflow to Sink Node)

(4) Import node :

$$\sum_{k=1}^T \sum_{j=1}^{n_t} \delta_{jk} I_k = X_m (\text{Inflow from Source Node}) \quad (3.13)$$

(5) Spill node :

$$\sum_{k=1}^T \sum_{j=1}^{n_t} \theta_j P_{jk} = X_s \text{ (Outflow to Sink Node)} \quad (3.14)$$

(6) Final storage node :

$$\sum_{j=1}^{n_r} \frac{S_{j,T+1}}{\Delta t} = X_f \text{ (Outflow to Sink Node)} \quad (3.15)$$

(7) Net balance node :

$$X_o + X_i - X_d + X_m - X_s - X_f = 0 \quad (3.16)$$

(8) Reservoir nodes : for the existing arcs in the network,

$$\begin{aligned} \sum_{i=1}^N Q_{ijk} - \sum_{i=1}^N Q_{jik} - \theta_j P_{jk} - \frac{S_{i,k+1}}{\Delta t} + \frac{S_{ik}}{\Delta t} \\ + \delta_j I_k + \alpha_{jk} - \beta_{jk} = 0 \end{aligned} \quad (3.17)$$

$$j = 1, 2, \dots, n_r \text{ and } k = 1, 2, \dots, T.$$

(9) Link junction nodes : for the existing arcs in the network,

$$\sum_{i=1}^N Q_{ijk} - \sum_{i=1}^N Q_{jik} - \theta_j P_{jk} + \delta_j I_k + \alpha_{jk} - D_{jk} = 0 \quad (3.18)$$

$$j = n_{r+1}, n_{r+2}, \dots, n_t \text{ and } k = 1, 2, \dots, T.$$

Table 3.5

## Definition of Terms used in AL-3 Model

<u>Network flow problem</u>		units <sup>a</sup>
$b_{ij}$	= Cost <sup>b</sup> of moving one unit of flow from node i to node j	$X(l^3/t)$
$L_{ij}$	= Lower bound on flow from node i to node j	$l^3/t$
$N$	= Number of nodes in the network	
$q_{ij}$	= Flow from node i to node j	$l^3/t$
$U_{ij}$	= Upper bound on flow from node i to node j	$l^3/t$
<u>Node balance equations</u>		
$D_{jk}$	= Demand at node j in time period k	$l^3/t$
$I_k$	= Rate of import <sup>c</sup> of water in time period k	$l^3/t$
$P_{jk}$	= Rate of spill from reservoir or link junction j in time period k	$l^3/t$
$Q_{ijk}$	= Flow between reservoir or link junction i and reservoir or link junction j in time period k	$l^3/t$
$S_{jk}$	= Storage contents of reservoir j in time period k	$l^3$
$\alpha_{jk}$	= Input to reservoir j (unregulated inflow) in time period k	$l^3/t$
$\beta_{jk}$	= (Demand + evaporation) from reservoir j in time period k	$l^3/t$
$\delta_j$	= 1, if j is an import node = 0, if j is not an import node	
$\theta_j$	= 1, if j is a spill node = 0, if j is not a spill node	
$\Delta t$	= Unit of each time period	

contd...

Table 3.5 (contd...)

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Subscripts

- i, j = Nodes  
k = Time period  
 $n_r$  = Number of reservoirs  
 $n_t$  = Number of reservoirs plus link junctions in each period  
T = Number of time periods

<sup>a</sup> l and t are used to designate unit of length and time respectively  
( $l^3/t$  = volume per unit time)

<sup>b</sup> X is given in relative cost units

<sup>c</sup> only one import node is considered.

---

The above model is used for the system of Basin BA. A description of this basin is given in the next chapter.

## CHAPTER 4

### ANALYSIS OF THE MODEL

#### 4.1 Introduction

Two major river basins in India are taken for study. Both the Streamflow Generation model and the Allocation model are applied to one of the basins BA and only Streamflow Generation model to the other basin BB. Basin BA has inflow data varying from 8 to 38 years at various nodes while for the basin BB, inflow data are available for an almost constant period of ten years. The hydrology of both these basins are governed by monsoons.

#### 4.2 Description and Modelling of Basin BA

The system consists of 15 reservoirs and 4 diversions and they are connected by river reaches. A description of this basin is given by Ramamurthy (1980). The same basin is analysed here with longer and more reliable data.

The river system of Basin BA in the network form is shown in fig. 4.1. Nodes represent reservoirs and diversions while the links represent the river or canal reaches. The details of the reservoirs and diversions are shown in Table 4.1. The details of power stations are shown in Table 4.2. Table 4.3 gives the details of system link connections, the bounds of the links and relative costs. The upper bounds of

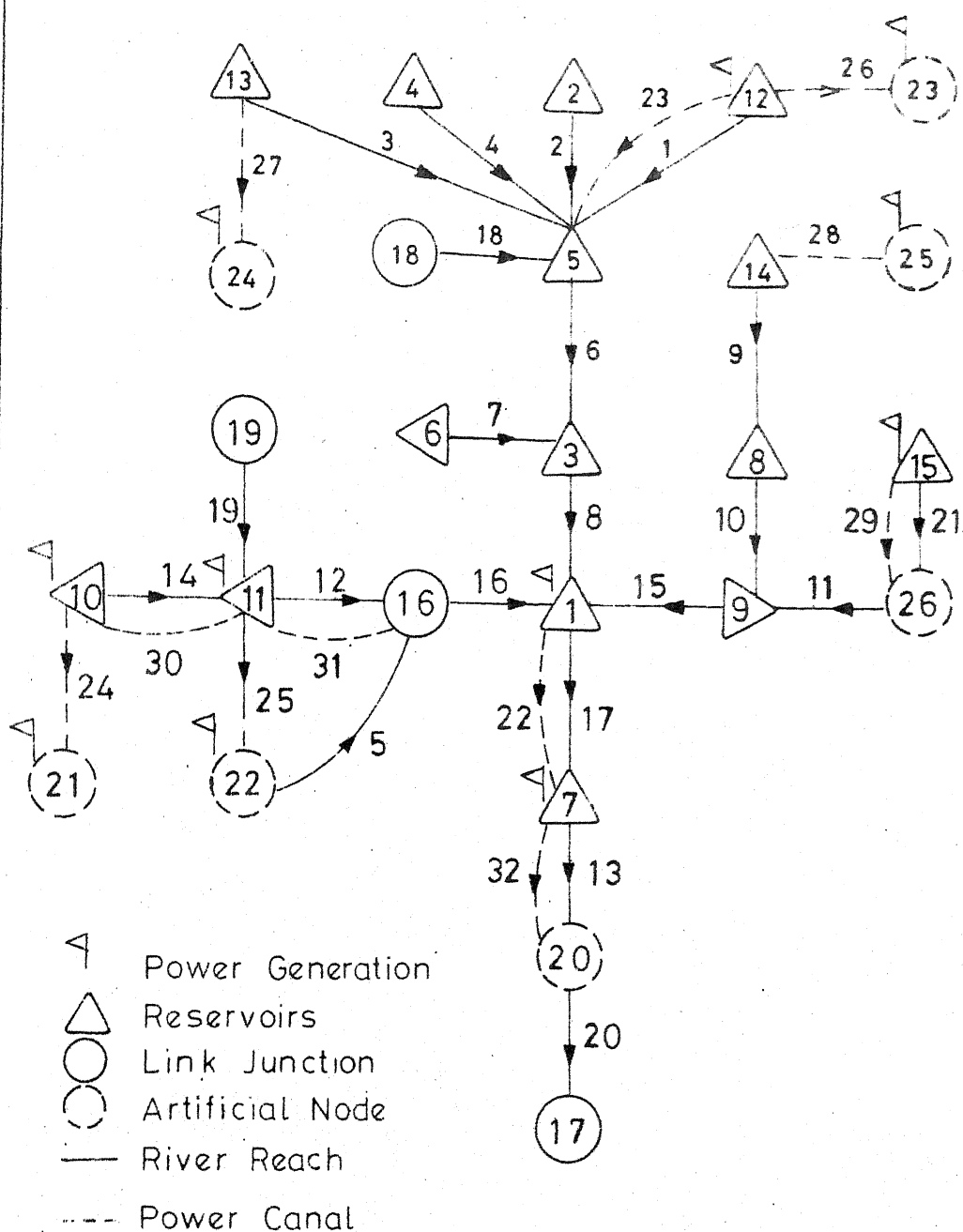


Fig. 4.1 Spatial network of basin BA



Table 4.1 Details of reservoirs and non storage nodes for Basin BA (values in 10 M.Cft)

Node No.	Reservoir Capacity		Annual demands	Mean flow	75 dependable flow
	Maximum	Minimum			
Storage nodes					
1	30800	15800	2532	3397	2741
2	3370	560	4240	8477	5947
3	3765	717	14840	2397	2104
4	5116	311	8835	9374	7319
5	4224	1182	7753	30909	27121
6	3774	713	4094	4799	3744
7	28200	19400	28086	2680	2194
8	11136	6000	8125	-	-
9	955	180	7038	16244	12777
10	7154	850	-	10132	7883
11	13300	1560	21760	18875	14798
12	9814	423	-	13116	11277
13	3960	940	600	8305	6556
14	11264	781	8531	30921	22931
15	3640	280	654	12098	9113
Non-storage nodes					
16	-	-	11279	10359	6990
17	-	-	30427	36421	28380
18	-	-	4200	11494	10085
19	-	-	1150	22750	16292
20	-	-	Artificial spill node	-	-
Artificial power nodes					
21	-	-	5990	-	-
22	-	-	4300	-	-
23	-	-	6750	-	-
24	-	-	2560	-	-
25	-	-	4259	-	-
26	-	-	4930	-	-
For the full basin			192933	252754	223007

Table 4.2 Details of power stations in Basin BA

Node No.	Link No.	Power MW	Capacity	Head		Remarks
			Max. flow (10 M.Cft)	Max.	Min.	
1 <sup>a</sup>	22	700	7775	343	311	Francis Turbine
12	23	50	550	237	105	Francis Turbine
21	24	15	790	77	18	Kaplan Turbine
22 <sup>b</sup>	25	70	710	201	155	Propeller Turbine
23 <sup>c</sup>	26	600	722	2047	1865	Pelton Turbine
24 <sup>d</sup>	27	20	320	137	80	Francis Turbine
25 <sup>d</sup>	28	280	450	1683	1659	Pelton Turbine
26 <sup>d</sup>	29	20	590	65	60	Kaplan Turbine
10	30	20	830	162	98	Francis Turbine
11	31	30	530	89	43	Propeller Turbine
7	32	400	9520	344	235	Francis Turbine

- a. Seasonal power station  
b. Includes canal power station and RHS dam power station  
c. Includes tail race power station  
d. Power station equivalent to all u/s power stations

Table 4.3 Details of link connections for  
Basin BA

Link No.	Node		Maximum flow in 10 M.Cft	Minimum flow 10 M.Cft	Relative cos
	From	To			
1	12	5	50,000	0	5
2	2	5	50,000	0	5
3	13	5	50,000	0	5
4	4	5	50,000	0	5
5	22	16	710	0	2
6	5	3	50,000	0	5
7	6	3	50,000	0	5
8	3	1	50,000	0	5
9	14	8	50,000	0	5
10	8	9	50,000	0	5
11	26	9	50,000	0	5
12	11	16	50,000	0	6
13	7	20	99,000	0	5
14	10	11	50,000	0	5
15	9	1	50,000	0	5
16	16	1	50,000	0	5
17	1	7	99,900	0	5
18	18	5	50,000	0	10
19	19	11	50,000	0	10
20	20	17	99,000	0	4
21	15	26	50,000	0	5
22	1	7	7,775	0	0
23	12	5	550	0	1
24	10	21	790	0	0
25	11	22	710	0	0
26	12	23	722	0	0
27	13	24	320	0	0
28	14	25	470	0	0
29	15	26	590	0	2
30	10	11	830	0	1
31	11	16	530	0	3
32	7	20	9520	0	0

canal links are equal to the maximum quantity of water that can be passed through turbines of power stations concerned.

#### 4.2.1 Demands

In India, irrigation projects are to be designed so as to meet 75 percent dependability and power projects to meet 90 percent dependability. In the present study irrigation demands have priority compared to power demands. The annual 75 percent dependable flows are reallocated to the various nodes in the system excluding the evaporation losses where evaporation losses are known. There are a number of minor schemes in the basin and their demands are added to the appropriate nodes. When the inflows during a year are less than 90 percent of the 75 percent dependable inflows, that year is termed as a dry year. For such years, demands are reduced to 90 percent of the planned demands. Similarly, when the inflows are greater by 10 percent of the 75 percent dependable flows in a year, that year is considered as wet year and demands are proportionately increased. The demands for the monsoon months are kept at average level since no additional reservoir storage space is available during these periods. The average year demand for all nodes is shown in Table 4.4. The monthly evaporation rates for the reservoir are shown in Table 4.5.

The monthly reservoir water spread areas and the heads acting over the turbines for the three hydrological states is shown in Table 4.6 and Table 4.7 respectively.

Table 4.4 Average Year Demands at Various Nodes for Basin BA

Node No.	Demand in 10 M.Cft.											
	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
1	215	402	329	321	276	229	174	177	158	44	38	146
2	329	538	463	316	274	323	323	323	337	351	351	351
3	1269	2381	1942	1900	1627	1354	1032	1048	928	257	228	865
4	725	1191	1191	1161	911	961	650	650	635	306	206	206
5	419	2867	2363	932	670	286	0	0	0	0	0	330
6	195	486	486	245	100	776	776	683	341	0	0	0
7	1355	5818	6123	5503	4925	3505	341	0	0	0	0	203
8	854	882	882	854	582	582	582	582	582	582	582	582
9	1479	2296	1609	969	276	111	83	81	39	19	15	80
10	Demand is zero											
11	1580	3430	3146	2494	2496	1791	1228	1265	1227	981	1111	1175
12	Demand is zero											
13	45	33	33	33	46	59	59	59	58	58	58	58
14	636	637	905	1004	861	820	748	752	536	494	576	566

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Demans in 10 M.Cft.											
June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	
37	250	206	81	57	24	0	0	0	0	0	
1193	2049	2034	1831	1694	711	355	338	280	314	243	
1990	6343	6980	4299	4974	2572	963	642	524	439	393	
199	1410	1273	542	318	208	39	26	22	25	38	
117	191	191	185	191	104	25	25	24	25	12	
Demand is zero											
520	755	787	747	761	560	389	368	329	328	369	
388	459	457	411	407	450	370	458	350	315	190	
602	611	585	566	568	570	573	577	582	586	595	
185	203	211	218	191	217	219	218	223	223	231	
335	382	394	393	358	355	346	334	345	325	343	
421	579	574	482	421	583	531	425	256	278	285	

Table 4.5 Evaporation Rates for Reservoirs

Node No.	Jun	July	Aug	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	April	May
1	.6950	.4630	.4630	.4630	.4630	.3090	.3090	.3090	.3090	.6950	.9270	.9270
2	.3698	.3698	.3698	.3698	.4215	.4215	.4215	.5364	.5364	.9502	.9502	.9502
3	.7090	.6670	.6250	.5830	.5000	.5000	.4170	.4170	.5830	.83301	.00001	.0000
4	.7500	.2500	.2500	.2500	.5000	.5000	.5000	.5000	.5000	.7550	.7500	.7500
5	.7090	.6670	.6250	.5830	.5000	.5000	.4170	.4170	.5830	.83301	.00001	.0000
6	.7600	.5000	.5000	.5000	.4000	.3333	.3333	.3333	.3333	.5100	.7600	.7600
7	.6950	.4630	.4630	.4630	.4630	.3090	.3090	.3090	.3090	.6950	.9270	.9270
8	.3667	.4167	.4167	.4167	.4167	.4167	.4167	.4167	.4167	.5833	.7500	.7500
9	.3667	.4167	.4167	.4167	.4167	.4167	.4167	.4167	.4167	.5833	.7500	.7500
10	.3333	.3333	.3333	.3333	.4167	.5833	.6667	.2500	.2500	.2500	.2500	.3333
11	.7500	.2500	.2500	.2500	.2500	.5000	.5000	.7500	.75001	.00001	.00001	.0000
12	.3698	.3698	.3698	.3698	.4215	.4215	.4215	.5364	.5364	.9502	.9205	.9502
13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 4.6 Average Monthly Surface Areas for Reservoirs

Node	Rule	Average monthly surface areas in M.sq.ft.											
		Jun	July	Aug	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
1	1	404	383	448	561	560	491	441	428	420	413	408	404
1	2	434	377	473	627	631	569	524	505	493	482	475	470
1	3	442	368	470	631	636	581	540	522	511	501	494	489
2	1	17	21	29	31	31	31	29	28	26	24	22	20
2	2	17	22	29	31	31	31	29	28	26	23	21	19
2	3	17	22	30	31	31	31	29	28	26	24	21	19
3	1	68	87	132	143	143	141	135	123	109	101	96	88
3	2	62	90	135	143	143	141	134	120	106	95	88	77
3	3	65	86	133	143	143	143	137	124	109	100	94	86
4	1	25	45	75	82	78	71	63	55	48	42	38	35
4	2	25	47	77	82	78	70	60	52	44	37	32	28
4	3	24	46	77	81	75	68	59	50	42	35	30	25

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Code	Rule	Average monthly surfaces areas in M.sq.ft.											
		June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
5	1	96	120	178	194	194	192	182	165	145	133	126	118
5	2	89	124	183	194	194	192	181	162	140	125	118	108
5	3	89	116	179	194	194	193	185	166	145	131	124	116
6	1	70	89	117	127	133	129	110	91	77	71	71	71
6	2	71	92	122	133	137	133	113	92	76	70	70	70
6	3	70	92	124	134	138	133	114	92	77	72	71	72
7	1	217	209	235	283	283	255	235	229	226	220	220	217
7	2	232	206	246	315	315	286	269	260	257	251	248	246
7	3	235	203	246	315	318	291	175	216	263	260	257	254
8	1	44	80	131	152	148	137	125	112	99	86	73	59
8	2	44	85	135	152	147	135	122	108	94	80	64	47
8	3	44	87	137	152	148	135	122	108	94	80	65	48
9	1	24	46	75	84	76	61	51	44	40	37	35	33
9	2	24	54	84	84	76	59	49	42	37	35	33	31
9	3	18	36	71	84	77	62	51	44	39	37	35	33

contd...

Table 4.6 (contd....)

Code	Rule	Average monthly surfaces areas in M.sq.ft.											
		Jun	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
0	1	28	58	102	120	123	102	110	97	85	70	53	38
0	2	28	61	108	126	126	104	112	99	83	67	47	31
0	3	26	60	110	126	126	123	113	101	87	70	50	33
11	1	119	203	346	401	400	384	353	337	301	264	223	183
11	2	118	209	359	407	400	382	359	330	290	247	199	156
11	3	110	195	354	407	404	392	373	347	311	273	232	191
12	1	39	71	111	119	120	115	111	101	89	74	54	40
12	2	39	71	111	119	120	115	111	101	89	74	54	40
12	3	39	71	111	119	120	115	111	101	89	74	54	40
13	1	000	000	000	000	000	000	000	000	000	000	000	000
13	2	000	000	000	000	000	000	000	000	000	000	000	000
13	3	000	000	000	000	000	000	000	000	000	000	000	000
14	1	000	000	000	000	000	000	000	000	000	000	000	000
15	2	000	000	000	000	000	000	000	000	000	000	000	000

Table 4.7 Average Monthly Power Head Rules

Code	Rule	Average monthly head in feet on turbines											
		June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
1	1	311	325	341	331	335	335	333	330	327	323	318	312
1	2	314	324	343	337	342	342	340	337	334	330	325	319
1	3	315	323	343	337	342	343	341	338	335	331	326	321
2	1	105	141	183	223	237	225	211	195	179	161	141	190
2	2	105	141	183	223	237	225	211	195	179	161	141	190
2	3	105	141	183	223	237	225	211	195	179	161	141	190
3	1	18	25	61	67	73	76	73	67	62	56	51	45
3	2	24	44	65	71	74	77	73	67	64	56	47	44
3	3	24	44	65	71	74	77	76	70	64	56	51	43
4	1	158	171	193	200	200	198	195	191	186	181	175	170
4	2	157	171	194	201	200	197	194	190	185	179	172	165
4	3	155	168	194	201	200	199	196	192	187	182	177	171
5	1	1865	1959	1970	2027	2047	2030	2011	1989	1967	1942	1915	1887
5	2	1865	1959	1970	2027	2047	2030	2011	1989	1967	1942	1915	1887
5	3	1865	1959	1970	2027	2047	2030	2011	1989	1967	1942	1915	1887

contd....

Table 4.7 (contd....)

Node	Rule	Average monthly head in feet on turbines											
		Jun	Jul	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
6	1	80	99	126	133	137	134	129	126	121	108	94	81
5	2	80	99	126	133	137	134	129	126	121	108	94	81
6	3	80	99	126	133	137	134	129	126	121	108	94	81
7	1	1659	1663	1677	1681	1683	1680	1674	1670	1666	1664	1664	1660
7	2	1659	1663	1677	1681	1683	1680	1674	1670	1666	1664	1664	1660
7	3	1659	1663	1677	1681	1683	1680	1674	1670	1666	1664	1664	1660
8	1	61	62	64	65	64	63	64	63	62	62	61	60
8	2	61	62	64	65	64	63	64	63	62	62	61	60
8	3	61	62	64	65	64	63	64	63	62	62	61	60
9	1	99	117	147	158	161	158	151	144	136	126	115	105
9	2	99	120	151	162	162	159	147	144	136	124	112	101
9	3	98	119	151	162	162	159	150	145	137	126	115	104

contd....

Table 4.7 (contd...)

Node	Rule	Average monthly head in feet on turbines											
		Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
10	1	46	59	81	88	88	86	83	79	74	69	63	58
10	2	45	59	82	89	88	85	82	78	73	67	60	53
10	3	43	56	82	89	88	87	84	80	75	70	65	59
11	1	244	253	280	317	296	293	241	250	247	241	238	235
11	2	256	250	293	344	329	323	271	277	277	271	265	262
11	3	259	247	293	344	329	326	274	283	280	274	271	268

#### 4.2.2 Determination of sub basin yields

The basin is subdivided into twelve sub basins. From the streamflow data and details of withdrawals available in each of the sub basin, the mean yields of each sub basin is estimated. An illustrative example is given below on the method adopted to obtain the mean yield of one of the sub basins. The sub basin chosen is shown in fig. 4.2.

Catchment area of the sub basin : 6939 sq. miles

Inflow details available :

Sl. No.	Stage discharge site	Catchment sq. miles	Mean yield TMC
1	SD1	2134	173.40
2	SD2	761	112.16
3	SD3	907	89.19

Estimation of sub basin yield :

Yield for 2134 sq. miles upto SD1 = 173.40 TMC

Yield for 761 sq. miles upto SD2 = 112.16 TMC

Yield for 907 sq. miles upto SD3 = 89.19 TMC

Estimate yield for T4 basin for 907 sq.miles as per SD3 (both lie in the same rainfall zone) = 89.19 TMC

Diversion at P1 = 86.98 TMC

Withdrawals at P2,P3 and P4 = 2.60 TMC

Estimated yield for remaining area of 2230 sq.miles at the rate of 18 M.Cft/sq.miles = 40.14 TMC

Mean yield of the sub basin = 593.66 TMC

say 590 TMC

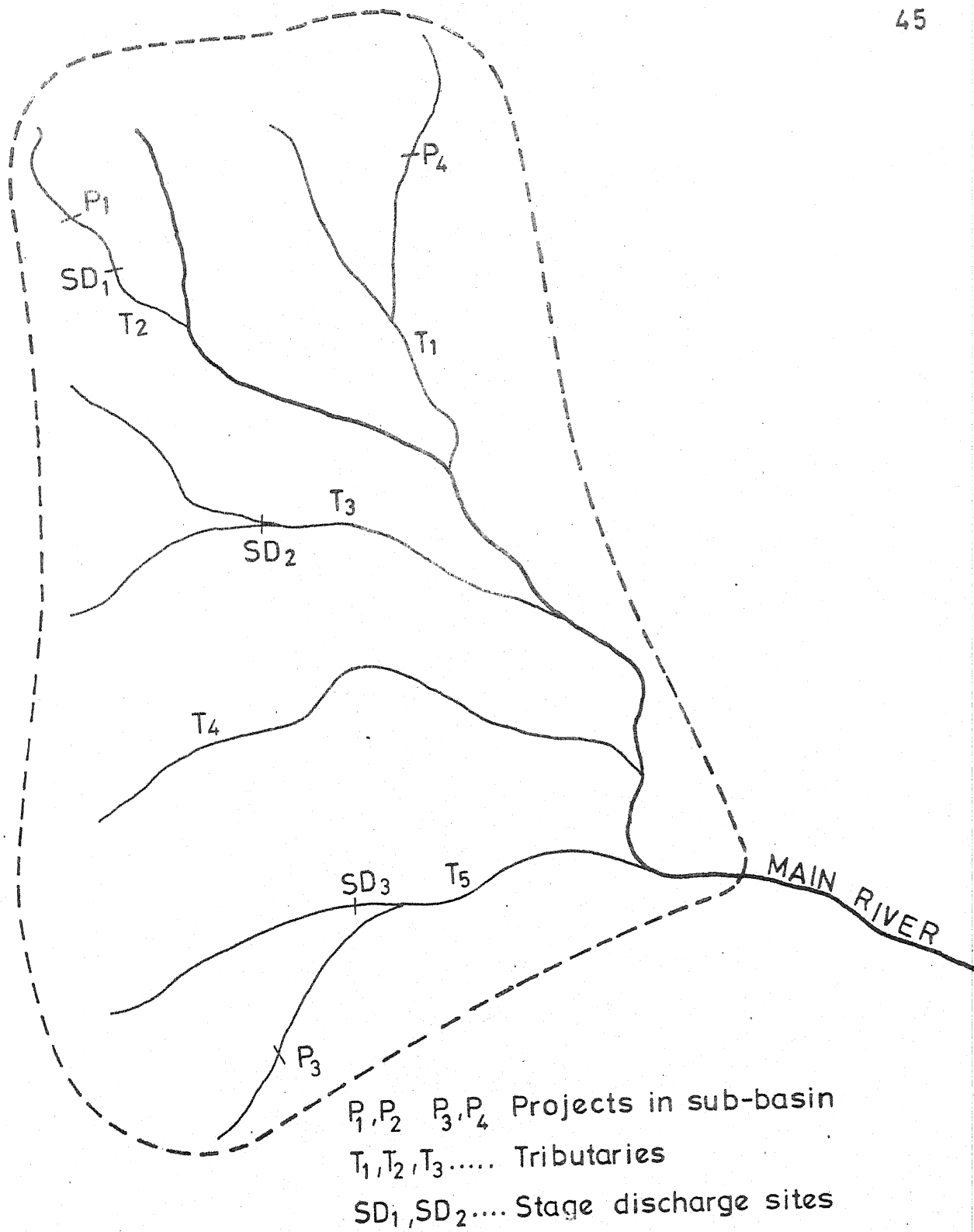


Fig.4.2 The sub basin SB A-1

The same procedure is adopted to calculate the mean yield of all the twelve sub basins. The mean yields of various sub basins are shown in Table 4.8.

#### 4.2.3 Streamflow Generation

##### 4.2.3.1 Data Base

Streamflows of sufficiently long periods are needed to simulate the system. For large river basins, it is difficult to collect the relevant concurrent streamflow data needed at representative points. Reliable measurement of data during the periods of peak flow is difficult and expensive. For the basin BA, gauged data are available for periods varying from 8 to 38 years, the longest being available at node 4.

For the present study, the virgin flows (the flow reaching a node from the independent catchments between the upstream nodes and the present node) are required at all the nodes of the system. The virgin flows are estimated at the various nodes by the following methods.

- 1) The procedure adopted to determine the mean annual flows for the sub basins as explained in article 4.2.2 is adopted to estimate the mean annual inflows at all nodes.
- 2) These mean flows are proportionately altered to maintain the total basin mean flow to be equal to the total mean yield of the whole basin.



Table 4.8

Yields of various  
sub basins of Basin BA

Sl.No.	Sub basin	Mean yield (T.M.C.)
1	SBA - 1	590.00
2	SBA - 2	60.00
3	SBA - 3	180.00
4	SBA - 4	70.00
5	SBA - 5	460.00
6	SBA - 6	70.00
7	SBA - 7	230.00
8	SBA - 8	540.00
9	SBA - 9	85.00
10	SBA - 10	65.00
11	SBA - 11	18.00
12	SBA - 12	80.00

- 3) For the node under consideration, the flows at the gauging station which is closest to the node are assumed to represent the dispersion of flows with respect to the mean monthly flows.
- 4) The annual mean flow at the gauging stations nearest to the node is proportionately altered such that this mean is equal to the mean calculated in step 2.
- 5) The effect of upstream releases, if any, are neglected in this estimate. The variation in the dispersion caused in the flows due to these upstream releases is also neglected. This simplification does not significantly affect the results of simulation since the total meanflow at the node is maintained.

Table 4.1 shows the annual mean flows for the various nodes. In the reconstituted flows at various nodes, negative values are assigned if the flow at any month is missing. These are recognized by the streamflow generation model and proper values are estimated to these points by multiple regression. This forms the data base for streamflow generation.

#### 4.2.3.2 Streamflow Generation of Basin BA

There are 26 nodes in the model of which nodes 20 to 26 are artificial nodes and hence virgin flows are all zero at these nodes. Due to limitation of computer memory size, only 8 nodes can be handled at a time for reconstituting the missing

flows and then generating the streamflows. Hence, groups of 8 nodes are taken at a time. Some important nodes are kept common for each set of generation of flows to maintain the cross correlation among various nodes. The nodes are grouped into four sets. The first set consists of nodes 14,12,4,10, 9,16,1 and 7. These are representative nodes taken from the upstream, middle and downstream portions of the river basin. From those, nodes 4,9,12 and 14 are carried forward and nodes 2,3,15 and 18 are added. For the third set, nodes 5,6 and 11 are considered along with 4,9,10 and 16 from previous sets. For the last set, nodes 3,7 and 19 are chosen with nodes 1,6,9 and 10 from the earlier passes. This forms the scheme for generation of synthetic flows.

Streamflows for 138 years were generated for all the nodes including the reconstitution of missing data in the 38 years of base period.

#### 4.2.3.3 Comparison of generated flows

The generated streamflows are statistically analyzed to determine their mean and standard deviations. These are shown in Table 4.9. The values of the statistical parameters for both recorded and reconstituted flows as well as for the generated flows compare favourably. The statistical parameters for the entire basin is shown in Table 4.10.

Table 4.9 Statistical Properties of Historical and Generated Monthly Flows  
for Basin BA (Flows are in 10 M.Cft)

Node No.	Flow	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Annual
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
							MEAN VALUES							
1	H	130	780	1111	554	572	139	46	14	17	11	7	18	3397
	G	137	786	1113	565	623	158	45	17	11	12	7	17	3497
2	H	283	3801	2387	1176	396	431	0	0	0	0	0	0	8477
	G	274	3974	2427	1176	436	331	2	0	0	0	0	0	8620
3	H	105	847	779	347	198	75	17	9	4	3	3	9	2397
	G	110	852	794	362	192	87	18	10	5	3	3	9	2444
4	H	626	3814	2617	1237	722	188	64	17	14	13	13	48	9374
	G	648	3925	2608	1257	559	178	70	22	13	9	10	53	9352
5	H	1355	10899	10031	4473	2558	974	250	122	61	41	39	126	30909
	G	1390	10943	10279	4490	2661	924	234	135	63	45	44	154	31362
6	H	235	1591	1105	716	556	202	81	74	81	31	39	83	4799
	G	256	1636	1176	717	534	204	75	78	79	29	41	92	4917
7	H	67	496	990	449	368	151	32	39	19	37	13	20	2680
	G	78	482	992	455	391	169	32	37	19	24	13	25	2718
8	H													
	G													

Flow is zero

Table 4.9 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
9	H	463	2587	4917	3959	2991	689	288	120	91	55	21	65	16244
	G	510	2505	4888	3866	3375	735	296	126	92	58	21	58	16529
10	H	727	3005	2762	1988	676	341	229	102	48	81	71	101	10132
	G	764	2853	2789	4058	647	327	223	108	48	79	70	94	12060
11	H	1060	5695	5532	2398	1824	1301	350	151	78	66	69	352	18875
	G	1060	5819	5304	2251	1801	961	356	143	74	65	71	352	18256
12	H	733	5503	4160	1825	449	103	54	109	36	35	75	32	13116
	G	684	5506	4165	1835	446	99	61	79	24	42	108	20	13089
13	H	471	2028	2540	1464	297	222	251	182	180	251	204	216	8305
	G	425	2098	2511	1665	289	226	257	188	187	258	212	231	8548
14	H	1247	8653	12641	6184	1533	307	122	71	42	45	37	39	50921
	G	1374	8381	12634	6663	1680	352	130	73	44	47	37	35	31451
15	H	400	3475	3733	1462	701	450	489	354	245	235	271	283	12098
	G	404	3726	3783	1477	747	457	490	355	251	235	274	294	12494
	H	279	906	2698	1869	2485	706	297	212	224	266	264	153	10359
	G	267	859	2740	1785	2571	764	301	219	227	268	276	151	10430

Table 4.9 (contd...)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
7	E	895	8522	8522	11438	6532	5765	1325	486	322	221	238	223	653	36421
	G	980	8909	8909	11496	5525	6019	1437	465	282	222	252	242	698	37331
8	H	504	4054	4054	3731	1664	951	362	85	45	23	15	14	46	11494
	G	540	4047	4047	3735	1724	910	301	81	45	24	14	13	45	11481
19	H	1432	7405	7405	7868	2979	1367	793	437	256	73	48	44	54	22756
	C	1561	7254	7254	9301	3075	1359	782	432	258	76	48	44	53	24242
<u>Standard Deviations</u>															
1	L	62	163	163	345	286	414	158	33	13	8	5	7	15	883
	C	65	179	179	345	337	488	195	39	14	8	5	8	16	1048
2	T	345	1639	1639	1573	1015	326	719	12	0	0	0	0	0	3314
	G	284	1948	1948	1854	839	326	534	5	0	0	0	0	0	3599
3	T	86	284	284	244	144	122	118	17	10	4	3	3	13	501
	G	84	272	272	231	124	102	128	11	8	6	4	3	9	452

contd ...

Table 4.5 (contd....)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
4		F	454	1166	988	1548	1273	247	103	15	21	49	50	199	3044
		G	438	1164	1047	1173	532	151	136	38	24	25	31	200	2174
5		H	1102	3650	3134	1849	1564	1523	213	126	56	37	35	167	5440
		G	592	4066	2980	1730	1350	938	180	116	61	44	43	175	5412
6		H	151	525	440	444	757	216	101	91	124	27	31	97	1328
		G	185	623	543	498	636	144	68	81	79	24	37	110	1480
7		H	45	240	364	173	195	189	16	44	6	101	5	32	538
		G	93	231	376	171	233	216	14	32	6	28	5	47	788
9		H	294	1347	2063	2476	2543	645	236	63	62	33	19	130	5374
		G	326	1324	2040	2454	3323	695	270	62	61	23	20	150	6321
10		I	590	1462	1183	4620	321	245	206	58	25	96	37	63	5960
		G	680	1330	1277	24683	306	199	174	50	24	88	33	56	25121
11		H	805	3172	2039	982	762	2451	179	109	68	64	66	267	5660
		G	837	4180	1852	811	753	922	192	88	55	53	59	218	5899

contd....

Table 4.9 (cont'd...)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
12	H	503	1772	1149	1097	369	62	61	412	107	49	95	65	2587
	G	468	1840	1299	1040	403	57	80	184	58	75	180	75	2567
13	H	428	1512	1350	1888	160	98	97	73	71	67	63	70	2415
	G	401	1496	1514	1803	169	99	92	78	74	64	64	87	2614
14	H	3343	3693	4754	3089	1180	394	77	33	28	41	45	50	10270
	G	2923	3683	4557	3595	1381	518	120	37	33	44	37	31	9528
15	H	179	1930	1671	913	505	127	68	62	40	34	41	62	3523
	G	160	2403	1579	860	595	129	66	65	42	33	37	62	3545
16	H	132	1087	3151	1295	1747	556	111	60	37	37	135	49	5367
	G	138	1128	2690	1269	1919	663	126	66	39	34	122	45	5300
17	H	830	6529	4234	3336	3696	1180	468	245	97	148	123	1508	11444
	G	883	7874	4185	3183	4233	1360	420	185	103	129	120	1424	13340
18	H	410	1358	1166	688	582	566	79	47	21	14	13	62	2396
	G	353	1480	1203	622	505	296	60	34	20	13	12	51	2342
19	H	1296	2740	6376	1522	420	226	114	177	22	4	9	37	9005
	G	1574	2777	10932	1714	408	210	108	174	21	5	9	32	12121

H - stands for historical and reconstituted flows for 38 years

G - stands for generated flows for 100 years



Table 4.10 Statistical properties of historical and generated annual flows for the entire Basin BA  
(All values in T.M.C.)

	Maximum flow	Minimum flow	Mean $\mu$	Standard deviation $\sigma$	75 percent dependable flow
Recorded and reconstituted flows for 38 years	4575	1380	2528	571	2230
Generated flows for 100 years	6153	1693	2588	657	2158

#### 4.2.3.4 Simulation of the system

The Allocation model is used to simulate the system. To reduce the memory size in the computer, a year is divided into five monthly periods and a last period consisting of the remaining months of the year. The maximum size of the network to be used consists of four years. Initial reservoir contents should be large enough to see that the simulation starts with a feasible solution. 105 years of generated flows are used for simulation.

Various priority values, benefits for the reservoir storage arcs and costs for the river and link arcs to achieve proper operation of the system are decided on the basis of a few preliminary computer runs. The simulation is done using 38 years of recorded and reconstituted flows so as to reduce the cost of computation time for the basin and the performance of the system studied. The final costs used for unit flows in the river and canal arcs are shown in Table 4.3. The results are presented and discussed in Chapter 5.

#### 4.3 Description and Modelling of Basin BB

The river system modelled in the network form is shown in fig. 4.3. All the reservoirs and diversions are shown as nodes and river reaches as links. The system consists of 21 reservoirs and 6 link junctions. Reservoirs 4,5,10,12 and 14 have power station below the respective dams. Provision is

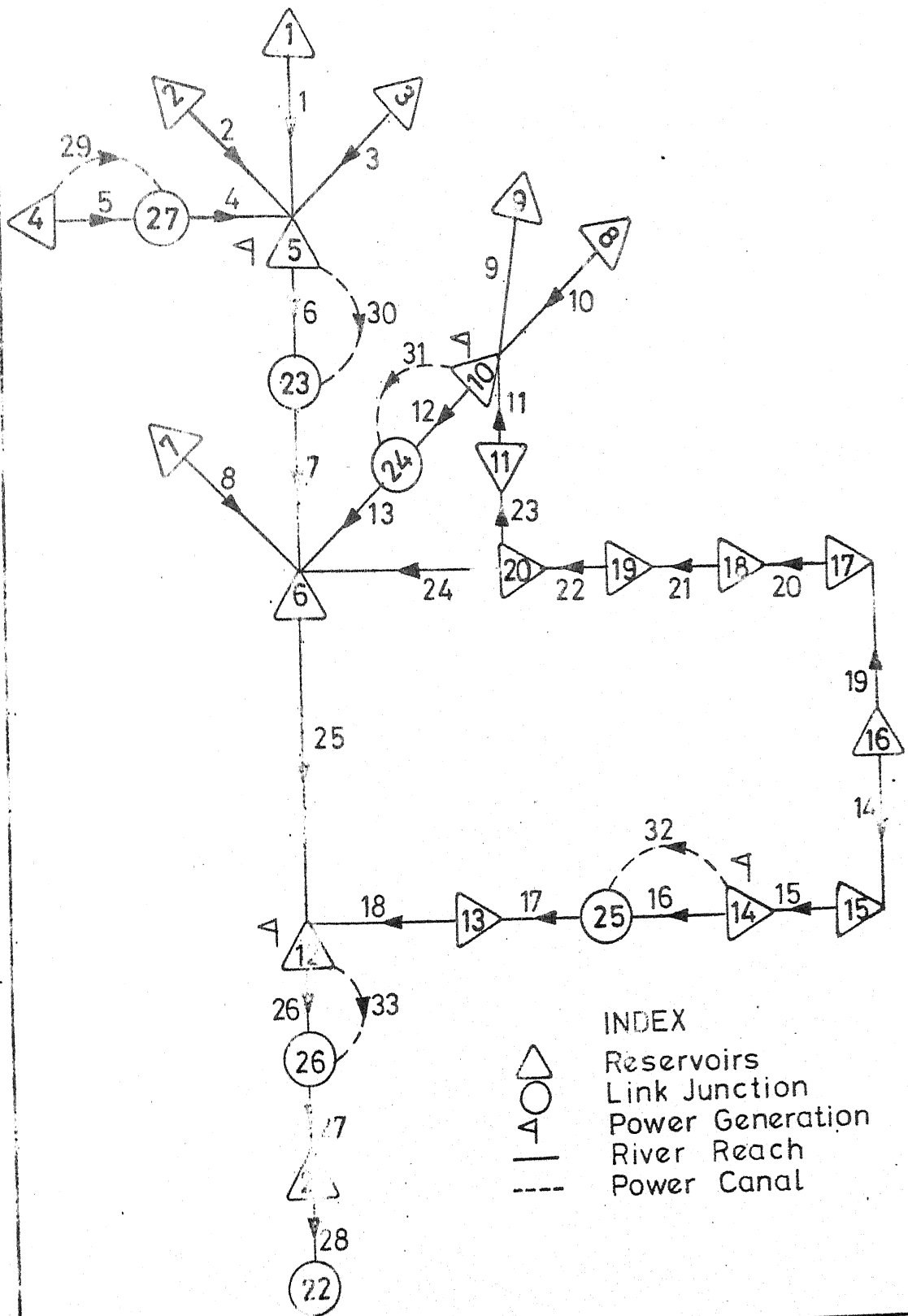


Fig. 4-3 Spatial network of basin BB

made at these reservoirs to pass the water through the power stations if the irrigation demand is low and this water flows to the downstream reservoirs after power is generated. Reservoir 9 is an equivalent reservoir representing all the upstream utilisation of water. From reservoir 11, water is diverted to two reservoirs viz. 10 and 20. Similarly, water is diverted to reservoirs 15 and 17 from reservoir 16.

Referring to the fig. 4.3, nodes 1 to 22 form the system nodes. Nodes 23 to 27 are added for calculation of power developed at various power stations.

Since only streamflow generation model is applied to this system, details other than that required for this model are not given there.

The mean yields of the various sub basins of this basin estimated on the same way as that of basin BA are shown in Table 4.11.

#### 4.3.1 Streamflow generation of Basin BB

The procedure adopted is same as that followed for the basin BA except that in this case 5 sets of stations are used. The first set consists of nodes 1,5,3,4,12,11,14 and 21. From these nodes 1,5,3 and 11 are carried forward and nodes 2,7,10 and 16 are added to form the second set. The third set consists of nodes 5,12,7 and 10 from the earlier sets together with nodes 8,9,20 and 6. Nodes 18,15 and 13 together

Table 4.11                      Yields of sub basins  
of Basin BB

Sl.No.	Sub basin	Mean yield (T.M.C.)
1	SBB - 1	179.49
2	SBB - 2	76.44
3	SBB - 3	101.84
4	SBB - 4	171.39
5	SBB - 5	198.91
6	SBB - 6	35.36
7	SBB - 7	175.59
8	SBB - 8	266.60
9	SBB - 9	812.83
10	SBB - 10	786.06
11	SBB - 11	706.92
12	SBB - 12	419.21

with nodes 1,5,20 and 6 form the fourth set. The last set consists of nodes 1,5,10 and 20 from earlier sets along with nodes 17,19 and 22.

The length of data available varies from 6 to 10 years and they are all concurrent.

The results of the flow generation are shown in Table 4.1 and it is found that the recorded and reconstituted data and the generated data agree statistically. The statistical parameters for the whole basin are shown in Table 4.13.

Table 4.12 Statistical properties of historical and generated flows for Basin BB  
(Flows are in 10 M.Cft.)

Flow (2)	June (3)	July (4)	Aug. (5)	Sept. (6)	Oct. (7)	Nov. (8)	Dec. (9)	Jan. (10)	Feb. (11)	Mar. (12)	April (13)	May (14)	Annual (15)
	<u>Mean Values</u>												
H	664	1649	4168	3907	1495	382	261	140	93	110	90	71	13027
G	600	1521	4089	4065	1564	396	285	146	97	124	100	71	13058
H	144	1271	1993	1984	456	34	36	15	10	4	1	1	5948
G	190	1320	1890	2065	502	39	43	17	12	6	1	1	6085
H	996	1114	2555	1711	681	203	145	111	108	81	120	182	8006
G	950	1105	2686	1867	731	214	160	120	125	103	151	247	8458
H	492	1604	3218	5992	2810	583	710	93	72	90	5	1	15668
G	485	1425	2703	7220	3216	622	955	231	123	107	14	0	17002
H	353	1181	3810	3870	1694	363	150	114	82	95	58	60	11828
G	328	1099	3868	4070	1840	379	165	128	87	108	68	59	12199
H	107	477	937	1008	297	76	50	27	17	32	10	7	3044
G	161	983	1810	1566	508	132	57	30	17	33	11	7	5314

contd...

Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
7	H	65	265	477	505	452	137	38	11	8	14	6	3	1980
	G	66	256	443	479	500	160	45	15	10	14	8	3	1998
8	H	1033	1655	4214	3251	1089	302	203	95	49	59	11	3	11964
	G	1417	1566	3946	3885	1172	541	207	97	51	65	14	4	12765
9	H	208	627	1000	1032	201	36	24	12	9	10	1	0	3159
	G	245	624	904	1110	206	35	26	13	10	11	1	0	3185
10	H	897	5629	10835	7834	2308	590	320	161	92	145	50	35	28895
	G	1072	5486	11024	8406	2368	560	340	168	98	141	52	35	29751
11	H	1207	13357	27653	15912	4653	1045	737	361	206	209	81	46	65466
	G	1366	12169	26550	16508	5191	1096	682	309	195	174	81	58	64378
12	H	519	3521	5543	4229	2151	1094	327	191	126	129	92	73	17995
	G	505	3736	5438	4245	2332	1400	337	197	132	142	93	71	18632
13	H	158	1076	1457	1156	614	145	104	66	50	49	48	50	4973
	G	163	1032	1527	1205	595	149	100	66	53	49	47	46	5032

contd....



Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
14	H	788	3593	3350	3060	1384	502	296	215	146	117	122	128	13702
	G	787	3676	3364	3110	1404	512	308	222	150	119	125	130	13906
15	H	804	529	402	673	673	777	892	965	940	924	731	693	9001
	G	780	501	427	782	723	789	889	990	956	943	759	734	9273
16	H	181	576	641	505	331	161	97	71	49	43	48	54	2757
	G	173	556	658	491	337	188	97	72	51	44	50	55	2771
17	H	1260	4608	6102	4959	2102	779	546	394	273	262	232	229	21747
	G	1281	4413	6311	5011	2070	800	543	392	273	274	241	237	21846
18	H	373	1507	1762	1440	618	225	159	114	79	76	68	66	6286
	G	384	1245	1763	1506	624	230	164	116	79	79	70	69	6329
19	H	328	2779	4516	3153	1268	334	183	114	69	54	50	48	12899
	G	359	2637	4819	3253	1217	371	182	115	70	57	54	50	13163
20	H	458	4372	6837	5078	2332	458	238	129	79	67	49	44	20140
	G	466	4162	6953	5233	2396	465	230	130	79	71	52	45	20290

contd....

Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
21	H	817	5875	9156	7685	7862	1419	601	360	238	241	312	154	34719
	G	827	6111	9238	7727	8281	1851	628	366	243	250	385	149	36057
22	H	1278	9371	17007	16879	12298	1885	967	580	383	388	503	245	61784
	G	1401	9282	17911	17920	10023	1854	992	578	385	410	677	247	61680
Standard Deviation														
1	H	735	1198	3698	4222	1079	132	255	71	44	96	76	73	8423
	G	761	1288	4127	5228	1116	125	244	71	46	93	93	94	10775
2	H	177	947	1370	1841	432	27	61	10	10	5	1	2	3132
	G	270	1054	1220	2181	591	30	82	12	13	9	2	3	3749
3	H	1157	962	2905	1753	413	147	102	101	121	100	150	250	6027
	G	1536	1223	3732	2306	432	144	117	111	120	133	251	475	7348
4	H	269	2346	4597	6729	1798	302	1491	59	116	254	7	3	11547
	G	266	2569	3968	9554	2050	286	1800	142	242	342	38	1	14467
5	H	351	902	4109	4917	1135	158	85	64	34	83	54	70	8811
	G	466	1015	5298	6296	1187	159	101	74	36	80	80	80	12498

contd....

Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
6	H	214	560	1048	1623	418	97	36	17	7	43	5	6	3335
	G	296	2819	5664	4958	1472	362	34	8	63	39	5	6	15140
7	H	100	235	565	662	671	120	44	11	8	29	7	4	1413
	G	88	256	561	707	592	148	54	19	8	34	18	5	1482
8	H	1308	859	4131	2922	956	197	159	53	32	89	11	4	7081
	G	3080	919	4468	4685	893	230	155	56	34	86	18	7	9961
9	H	264	692	1126	1389	238	39	16	7	8	12	2	1	2296
	G	570	688	910	1721	238	29	19	8	7	14	2	1	2663
10	H	1056	3636	5036	3902	2483	560	230	89	51	146	36	31	11822
	G	2424	3788	5250	3541	1980	356	304	84	47	128	37	27	11821
11	H	1803	8391	10459	9599	4178	803	577	215	79	163	41	40	24775
	G	2913	8161	10646	9617	4378	751	607	151	72	105	37	146	27351
12	H	567	2396	3134	3028	1393	1664	109	77	46	66	36	20	7641
	G	707	3265	3019	3184	1378	2565	116	77	48	78	42	20	9485

contd ...

Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
13	H	106	614	954	453	532	42	44	32	34	36	36	35	1437
	G	121	562	1120	469	398	38	35	32	56	40	48	42	1910
14	H	493	1906	430	1011	188	104	58	31	17	15	33	32	2239
	G	577	2333	401	971	196	99	61	30	15	15	29	33	3025
15	H	371	189	322	436	290	186	252	218	172	223	403	422	1843
	G	368	181	349	563	342	198	235	272	168	202	356	477	1692
16	H	77	269	253	170	67	79	10	7	6	6	13	10	434
	G	66	267	219	145	68	148	10	8	6	6	13	10	539
17	H	918	2381	3366	1752	1409	152	116	53	30	58	44	66	5786
	G	911	2100	3707	1723	1601	161	120	53	27	71	44	57	6398
18	H	268	720	998	510	409	44	34	15	9	17	13	20	1735
	G	275	594	905	503	362	42	32	13	8	19	12	18	1514
19	H	195	1787	3467	1228	845	149	52	23	12	15	17	21	5104
	G	201	1676	4424	1246	954	236	53	22	11	18	17	18	6371

contd ...

Table 4.12 (contd....)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
20	H	285	2859	3629	1953	1893	304	54	29	17	22	22	30	7599
	G	317	2685	3303	1950	1772	382	49	26	15	21	22	37	6884
21	H	766	5504	7138	4763	11167	1109	192	106	61	109	376	62	15199
	G	1017	4597	9212	5021	8926	2056	212	99	60	96	1200	62	21661
22	H	1178	5586	9604	8418	18140	1294	309	170	99	175	605	101	19486
	G	2047	5850	10081	8572	9168	971	364	156	96	205	2205	109	25994

Table 4.13 Statistical properties of historical  
and generated flows for the entire  
Basin BB  
(Values in 10 M.Cft.)

	Maximum flow	Minimum flow	Mean	Standard deviation	75% det dable f
Recorded and reconstituted flows for 10 years	576318	217326	374988	126706	276680
Generated flows for 100 years	993460	119717	383173	166803	269333

## CHAPTER 5

### RESULTS AND DISCUSSIONS

#### 5.1 Introduction

##### 5.1.1 General

Optimization and simulation techniques capable of analyzing in detail large water resources system are very valuable to planning engineer. The real complex system is represented by model and the above techniques are applied to it. While models cannot make decisions, they can provide valuable information concerning the construction and operation of a proposed set water resources projects.

##### 5.1.2 Streamflow generation

Historic records of streamflow are too short to include all possible patterns of droughts and floods. The generation of synthetic streamflows provides longer sequences which permit more extensive analysis of the consequences of low and high flows on the output of water resources systems than do techniques which use only the historical record. In this study, a monthly streamflow generation model developed by Hydrologic Engineering Centre is used to generate streamflow. The inflow data used varied from 7 years to 38 years for basin BA and 7 to 10 years for basin BB. On the basis of this information, 138 years of flow has been generated for basin BA and 110 years for basin BB. The program takes about 50K memory

the DEC 1090 system. The CPU time for streamflow generation for basin BA was about 2 minutes while for basin BB, which has a larger system, it was about 5 minutes. The generated flows compare statistically well with the historical data as seen from Tables 4.10, 4.13. The method adopted here, thus, can be used to get the data base for simulation studies.

### 5.1.3 Optimization

The generated streamflows are used in the optimization study. The optimization technique used uses the out-of-kilter algorithm which is one of the most efficient ways of solving a network flow model. The program developed by the Texas Water Development Board is used with suitable modification.

105 years of generated inflows are used. 7 runs were taken in the DEC 1090 computer system, using 15 years for each run. The CPU time for each run is about 14 minutes and the model requires about 70K memory. The results of the simulation presented in this report include the demand deficits at each node, the surplus water at spill nodes, annual power generated and end of the month storages. The frequency analysis of the demand deficits give a method of evaluation of individual projects with respect to their expected performance. The statistical analysis of the spills indicate the quantity of water available after satisfying the in-basin demands. The end of the month storages are used to develop rigid operating rules for the reservoirs.



## 5.2 Results of Studies and Conclusions

In India, irrigation projects are planned such that demands are to be met with 75 percent dependability i.e. 100 percent of the demand should be met at least 75 percent of the time. The frequency analysis of demands met or exceeded is shown in Table 5.1. Table values indicate the percentage of time the stated frequencies of demand being met or exceeded. It is found from the table that nodes 4 and 6 have large deficits. For node 4, only 30.48 percent of time, 95 percent of demand is met or exceeded and for node 6, only 39.05 percent of time, 95 percent of demand is met or exceeded. This shows that the two projects represented by these two nodes have not been properly planned. The demand to be met is much higher than the inflow available at these two sites. For all other reservoirs the design criterion has been met or nearly met. Fig. 5.1 shows a plot of percentage of demand met or exceeded with the percentage of time for two selected nodes.

Table 5.2 shows the statistical properties of spills. The amount of water available for out of basin transfer at nodes 1 and 17 after meeting the in basin demands on a monthly basis is indicated in the table. At node 1, it is found that surplus flows occur only during monsoon months while at node 17, a small quantity of water flows out even during non-monsoon months. This is due to water coming from the sub-basins in between nodes 1 and 17. As seen from the table,

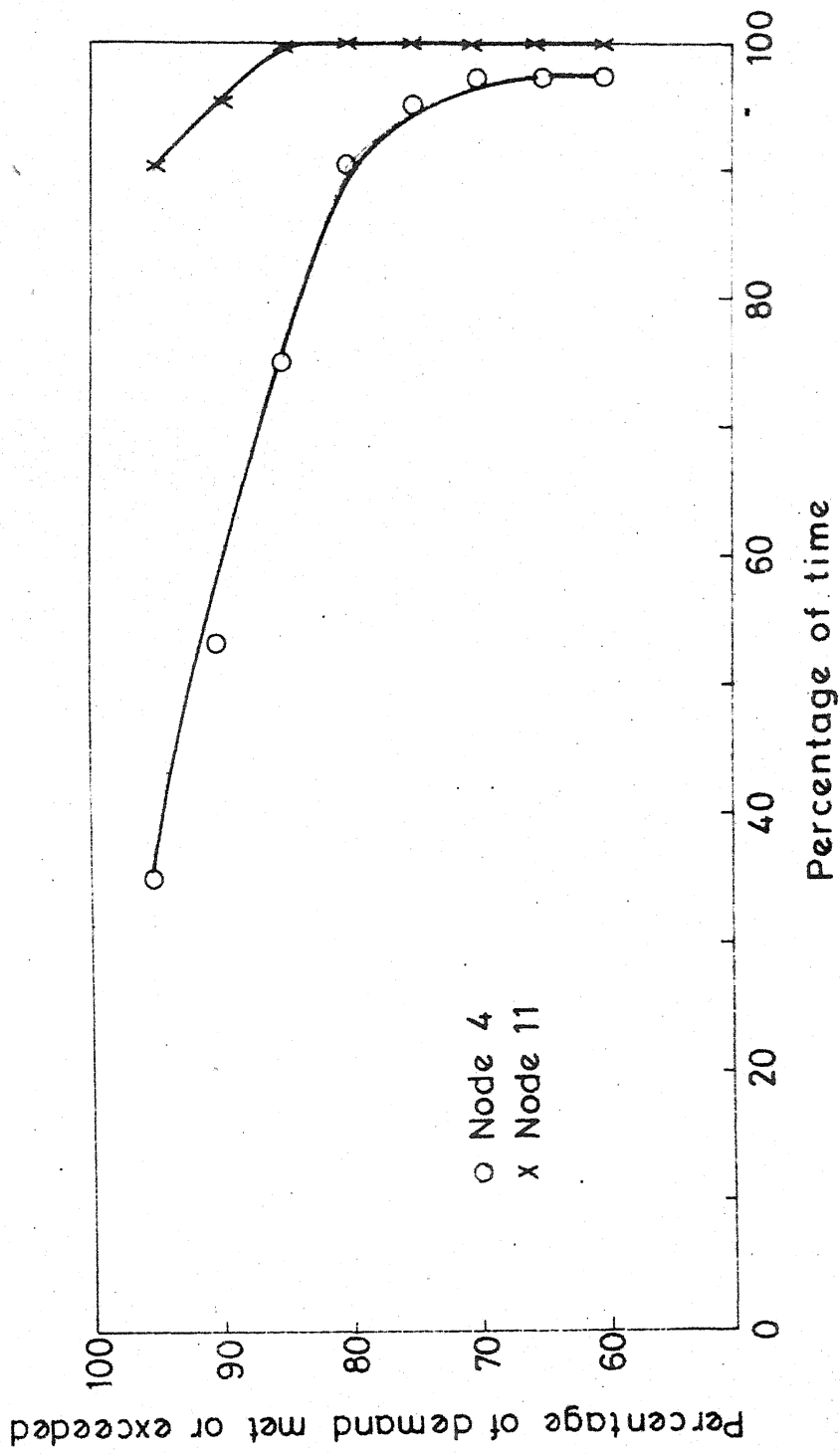


Fig. 5-1 Frequency analysis of demand met at selected nodes

Table 5.1 Frequency Table

Node	Percentage of demand met or exceeded									
	95.00	90.00	85.00	80.00	75.00	70.00	65.00	60.00		
1	95.24	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
2	91.43	93.33	96.19	96.19	97.14	99.05	99.05	99.05		99.05
3	98.10	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
4	30.43	53.33	75.24	90.48	95.24	97.14	97.14	97.14		97.14
5	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
6	39.05	54.29	70.48	90.48	94.29	96.19	97.14	97.14		97.14
7	95.24	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
8	93.33	98.10	100.00	100.00	100.00	100.00	100.00	100.00		100.00
9	99.05	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
11	90.48	95.24	100.00	100.00	100.00	100.00	100.00	100.00		100.00
13	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
14	95.33	98.10	100.00	100.00	100.00	100.00	100.00	100.00		100.00
15	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00

contd....

Table 5.1 (contd....)

Node	Percentage of demand met or exceeded									
	55.00	90.00	85.00	80.00	75.00	70.00	65.00	60.00		
16	81.90	91.43	96.19	99.05	100.00	100.00	100.00	100.00		100.00
17	98.10	99.05	99.05	100.00	100.00	100.00	100.00	100.00		100.00
18	67.62	93.33	98.10	99.05	99.05	99.05	100.00	100.00		100.00
19	91.43	94.29	95.24	99.05	99.05	99.05	99.05	99.05		99.05
21	83.81	93.33	96.19	97.14	97.14	97.14	97.14	97.14		97.14
22	80.00	91.43	96.19	98.10	98.10	98.10	98.10	98.10		98.10
23	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
24	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00
25	93.33	98.10	100.00	100.00	100.00	100.00	100.00	100.00		100.00
26	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00		100.00

Note : The demands at nodes 10 and 12 are zero. Node 20 is an artificial spill node.  
 Table values indicate the percentage of time the stated frequencies of demand being met or exceeded.

Table 5.2 Statistical properties of spills  
(Values in 10 M.Cft)

Node No.	Item	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Annual
1	Mean	14029	8792	4364	128	70	0	0	0	0	0	0	0	27382
	Standard Deviation	11582	16080	14896	965	719	0	0	0	0	0	0	0	32588
	Maximum	54276	82798	91060	8777	7372	0	0	0	0	0	0	0	182043
	Minimum	0	0	0	0	0	0	0	0	0	0	0	0	0
17	Mean	78	3148	4421	2521	1685	308	12159	190	204	204	175	120	25213
	Standard Deviation	376	4948	3239	2934	2729	956	8368	23	26	37	65	58	16751
	Maximum	3129	26340	12055	13957	12684	5347	38424	218	218	218	218	218	77747
	Minimum	0	0	0	0	0	0	0	0	0	0	0	0	0

an average annual surplus of 273.82 TMC is available at node 1 for out of basin transfer.

Another output of the optimization model is the end of the month storages for all the reservoirs in the system. These are classified for the three hydrological states namely dry, average and wet. The mean value calculated for each month and each hydrological state form the rigid rules for reservoir operation. Table 5.3 shows these rigid rules for all the reservoirs in the system. It is seen from the table that the dry year operating rule during monsoon period is higher than that of average and wet years. This indicates the carryover storage brought from the end of the previous year. During non-monsoon period, the dry year rule is at the lowest indicating that the reservoir storage has to be lowered to meet the demands. The wet year rule during monsoon is the lowest indicating very small quantity of carry-over storage. During non-monsoon, it is the highest indicating storage preserved for future use. These trends are observed in most of the reservoir operating rule developed here. Plots of rigid operating rule for node 1 and 4 are shown in Fig. 5.2(a) and (b).

The results of the study also contain the annual power generated for each of the power stations in the system. The annual power generated is statistically analysed to find the mean and standard deviation and the results are presented in Table 5.4.

Table 5.3 Rigid rules for Reservoir Operation

Reser- voir No.	Rule	End of month reservoir contents (10 M.Cft.)											
		June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	Dry	22216	23423	26524	25826	25084	24741	24380	24006	23651	23398	23147	22807
	Avg	16768	21196	27885	29231	29104	28818	28539	28238	27955	27780	27607	27343
	Wet	15869	17484	24449	28817	30061	29951	29831	29679	29539	29493	29438	29316
2	Dry	863	2550	2730	3046	2996	2765	2457	2145	1820	1483	1146	809
	Avg	823	2340	2940	3264	3196	3019	2728	2419	2079	1725	1354	999
	Wet	603	1400	2777	3245	3280	3159	2932	2665	2338	1982	1625	1253
3	Dry	1485	2744	2901	2978	3163	2465	2027	1589	1292	1427	1675	1357
	Avg	1318	2358	3262	3380	3471	2790	2367	1929	1647	1916	2274	2009
	Wet	1078	1575	2874	3354	3717	3093	2723	2329	2053	2436	2845	2638
4	Dry	1089	3479	4557	4499	4092	3400	2879	2324	1785	1503	1302	1139
	Avg	1098	3581	4552	4371	4055	3363	2807	2215	1632	1347	1151	969
	Wet	776	2948	4339	4493	4257	3572	3055	2473	1892	1595	1393	1258

contd....

Table 5.3 (cont'd ...)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
5		Dry	1793	2589	3001	3042	3231	3595	3591	3476	3313	3076	2779	2308
		Avg	1976	2974	3784	3894	3765	3800	3764	3693	3562	3381	3165	2721
		Wet	1251	2029	3424	3814	4051	4024	3981	3917	3822	3703	3568	3259
6		Dry	1184	2149	2705	2830	3138	2588	1950	1413	1215	1208	1213	1232
		Avg	1401	2363	2747	2964	3150	2617	1952	1370	1122	1113	1108	1146
		Wet	1096	2055	2583	3033	3390	2873	2216	1639	1364	1346	1340	1367
7		Dry	25804	24482	24779	26201	24322	21218	20911	20933	20936	21002	20999	20822
		Avg	26107	25586	26672	27585	26765	23269	22852	22781	22695	22612	22519	22236
		Wet	21077	21901	24263	26145	27564	24329	24075	24166	24236	24310	24374	24241
8		Dry	6547	7641	9338	10435	10512	9967	9422	8877	8332	7787	7243	6698
		Avg	6326	7509	10555	11038	10597	10119	9641	9162	8684	8206	7727	7249
		Wet	6061	6714	10274	11066	10861	10464	10067	9670	9273	8875	8478	8081
9		Dry	286	533	757	726	688	837	871	871	884	903	912	899
		Avg	253	548	799	843	793	845	876	872	875	871	843	845
		Wet	196	326	779	812	898	930	932	927	918	901	875	847



Table 5.3 (contd....)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
10		Dry	2292	4628	5588	5849	5347	4773	4267	3733	3190	2660	2121	1897
		Avg	2804	4410	5631	5807	5442	4863	4372	3817	3243	2703	2121	1860
		Wet	1605	3483	5839	6481	6289	5747	5263	4648	4002	3387	2715	2381
11		Dry	2375	7938	11671	12130	11481	10233	9257	7858	6411	5172	3965	3068
		Avg	3173	7954	11848	12341	11662	10775	9755	8245	6687	5313	3982	3054
		Wet	1873	4372	11487	12614	12449	11447	10586	9243	7709	6365	5049	4136
12		Dry	1174	5272	7414	7664	7035	6236	5377	4515	3644	2800	1976	1101
		Avg	1163	5554	8336	8970	8433	7472	6441	5464	4409	3370	2399	1333
		Wet	694	4456	8197	9484	9355	8272	7155	6036	4867	3704	2579	1392
13		Dry	1580	2447	3161	3071	2604	2509	2432	2237	2032	1930	1756	1609
		Avg	1775	2805	3339	3662	3357	3197	3056	2851	2638	2499	2311	2145
		Wet	1154	1724	3263	3918	3608	3473	3372	3230	3076	2987	2847	2739
14		Dry	3094	5576	8547	8815	8058	7076	6107	5122	4298	3537	2742	1901
		Avg	2214	5771	9985	10811	9818	8668	7512	6326	5322	4382	3336	2301
		Wet	1093	2806	8881	10878	10852	9849	8671	7427	6352	5344	4225	3106
15		Dry	618	1761	3104	3250	2939	2568	2473	2159	1881	1580	1286	1014
		Avg	733	2102	3167	3421	2974	2543	2441	2101	1812	1502	1209	924
		Wet	432	979	2836	3416	3363	2960	2868	2588	2394	2146	1928	1720

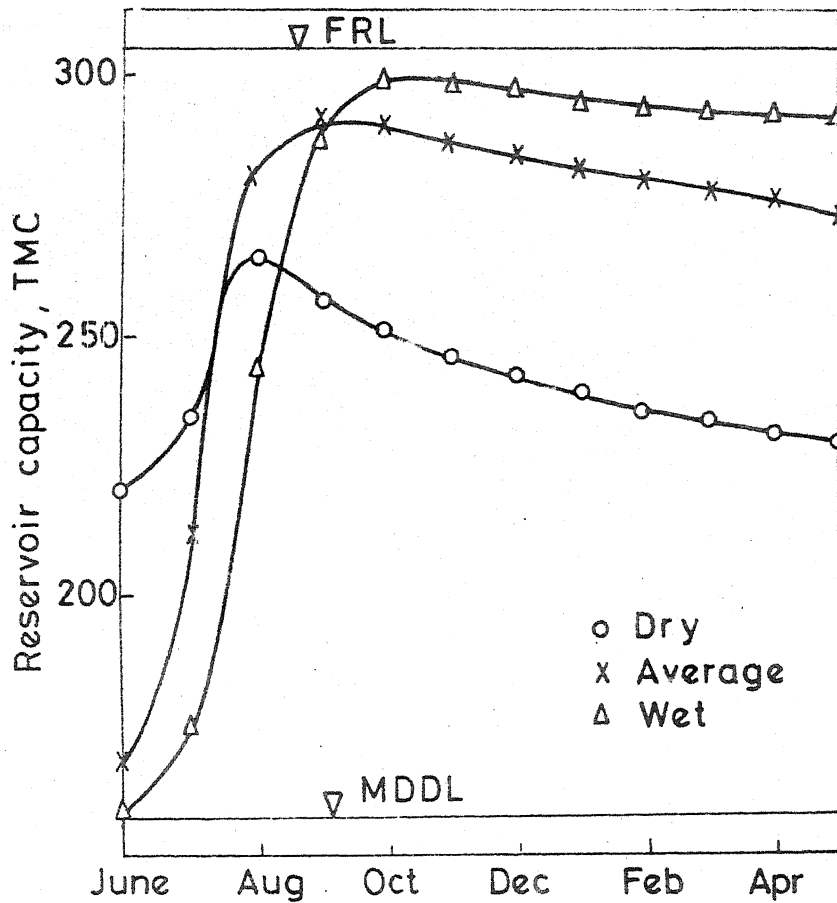


Fig.5.2 (a) Rigid operating rule for Node 1

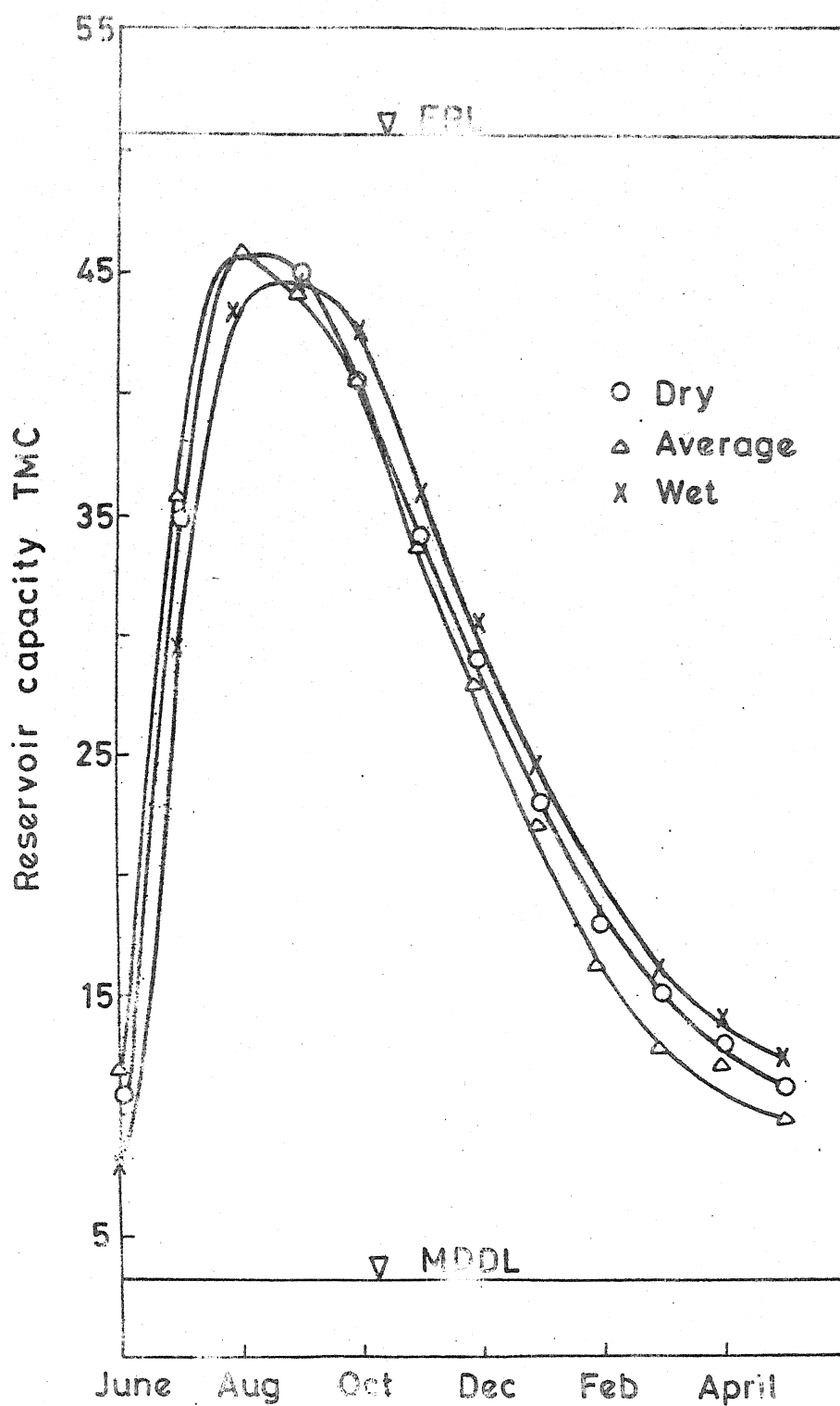


Fig.5.2(b) Rigid operating rule for Node-4

Table 5.4 Statistics of annual power generated  
(all values are in 10 kilowatts)

Node No.	Power Station Link No.	Mean	Std. Dev.
1	22	24447.30	2486.62
12	23	1689.71	588.69
21	24	765.62	48.55
22	25	2864.93	540.02
23	26	30962.54	1400.02
24	27	654.51	28.40
25	28	15896.06	998.26
26	29	947.21	87.63
10	30	870.56	523.49
11	31	372.23	193.74
7	32	4551.73	2183.90

In conclusion, it may be stated that large water resources system can be analyzed by using the technique followed in this study. The reliability of the results of such studies depend not only on the proper representation of the physical system by a model, but also on the use of reliable data. The model requires to be updated periodically to reflect the changes that take place in the basin.

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